

Shliomis model based magnetic fluid lubrication of a squeeze film

in rotating rough curved circular plates

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ABSTRACT

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An endeavor has been made to analyze the performance of a ferro fluid based squeeze film in rotating rough curved circular plates resorting to Shliomis model. The stochastic model of Christenson and Tonder has been deployed here to evaluate the effect of surface roughness. The associated stochastically averaged Reynolds type equation is solved to obtain the pressure distribution culminating in the calculation of load carrying capacity. The results show that Shliomis model based ferrofluid lubrication is relatively better as compared to the Neuringer-Rosensweig model for magnetic fluid lubrication of this type of bearing system. The adverse effect of roughness can be reduced considerably at least in the case of negatively skewed roughness with a suitable choice of curvature parameters. Further, by making a proper choice of rotational parameter .it is observed that this type of bearing system supports some amount of load even when there is no flow, unlike the case of conventional lubricants.

1.1. 1. Introduction

The review of Shliomis [1] discussed briefly the methods of preparation and stability problems of magnetic colloids. This review paper summarized the results of theoretical and experimental investigations of the effect of a magnetic field on the equilibrium conditions and on the character of the motion of the suspensions. Consideration was given to various effects caused by rotation of the particle, anisotropy of the viscosity and of the magnetic susceptibility, entrainment of the suspension by a rotating field and dependence of the kinetic coefficients on the field intensity. Patel [2] investigated the effect of velocity slip on the behaviour of a squeeze film between two circular disks under the application of a uniform magnetic field. Here it was shown that for a better performance of the bearing system the slip parameter deserved to be minimized. Verma [3] considered the squeeze film lubrication of a magnetic fluid between two approaching rectangular surfaces in the presence of an externally applied magnetic field oblique to the lower surface. The performance of the magnetic fluid based squeeze film was definitely enhanced and the time for the upper plate to come down was found to be longer than the viscous squeeze film. Bacri et al. [4] noted that the magnetic energy was partially transformed into the angular momentum of the magnetic particles which in turn, was converted into a hydrodynamic motion of the fluid. Zahn [5] proposed magnetic fluid based micro/nano electro mechanical system that used 10 mm diameter magnetic particles with and without a carrier fluid. The study of Lu et al. [6] was devoted to the effects of fluid inertia in magneto hydrodynamic annular squeeze films. It was shown that the inertia correction factor in the magneto hydrodynamic load carrying capacity was more pronounced with large Hartmann numbers. Lin et al. [7] dealt with the influences of convective fluid inertia forces in magnetic fluid based conical squeeze film plates in the presence of an external magnetic field considering Shliomis ferromagnetic fluid model. This study suggested that the magnetic fluid based conical plates operating with a large value of the inertial parameter of fluid inertia performed in a better way, as compared to the non inertia non magnetic cases. Lin et al. [8] considered the effects of viscosity-pressure dependency on the non Newtonian squeeze film in parallel circular plates. According to the results, the influences of viscosity-pressure dependency raised the load carrying capacity.

Gupta et al. [9] investigated the effect of rotational inertia on the squeeze film load between porous annular curved plates. It was shown that the load carrying capacity decreased when the speed of rotation of the upper disk increased up to certain value. It was noted that the load capacity could be enhanced without altering the speed of rotation by increasing the upper plate's curvature parameter. Prakash and Tiwari [10] analyzed the effect of surface roughness on the squeeze film performance between rotating porous annular disks with arbitrary porous wall thickness. Bhat and Deheri [11] theoretically studied the squeeze film behaviour between two annular disks when the upper plate with a porous facing approached the parallel lower disk in the presence of a magnetic fluid lubricant. This study established that the performance of the bearing with the magnetic fluid lubricant was much better than that with the conventional lubricant. Prajapati [12] dealt with the squeeze film behaviour between rotating porous circular plates with a concentric circular pocket. The introduction of the pocket reduced the load capacity of the bearing. Bhat and Deheri [13] theoretically analyzed the effects of magnetic fluid on the action of a curved squeeze film existing between two circular disks when the upper porous disk moved normal to itself and approached the impermeable and flat lower disk. The load carrying capacity and response time increased with increasing magnetization. But the effects due to magnetization were independent of the curvature of the upper disk.

Patel and Deheri [14] considered the magnetic fluid lubrication of a squeeze film between porous circular plates with concentric circular pockets and evaluated the effect of surface roughness. The transverse surface roughness decreased the load carrying capacity while the magnetization raised the load carrying capacity. Further, the pocket size significantly affected the bearing system. Deheri and Patel [15] made an effort to study the effect of magnetic fluid lubricant and sealing of boundary for the squeeze film performance between two porous circular disks. It was shown that the sealing of the boundary increased the load carrying capacity significantly. Of course, the magnetic field resulted in higher load carrying capacity. Abhangi and Deheri [16] considered the numerical modelling of magnetic fluid based squeeze film performance between rotating transversely rough curved circular plates. The negatively skewed roughness increased the already enhanced load carrying capacity due to magnetization. Besides, for a suitable choice of curvature parameters the combined effect of variance (-ve) and negatively skewed roughness was significantly positive. Naduvinamani et al. [17] analyzed the effect of surface roughness on magneto hydrodynamic couple stress squeeze film lubrication between circular stepped plates. It was observed that the applied magnetic field increased the load carrying capacity and lengthened the squeezing time while the roughness significantly affected the bearing system. Recently, Shimpi and Deheri [18] studied the ferrofluid lubrication of a squeeze film in rotating curved rough porous circular plates and evaluated the deformation effect. It was shown that the deformation turned in an adverse effect on the performance of the bearing system. However, this negative performance was minimized by the magnetic fluid lubrication for a large range of curvature parameters.

Here it has been proposed to study the effect of transverse surface roughness on the Shliomis model based magnetic fluid lubrication of a squeeze film in rotating rough curved circular plates.

2. Analysis

The bearing, which is displayed in Figure-1, consists of two plates, each of radiuses a. The lower and upper plate rotates with angular velocities Ω_1 and Ω_u respectively, about the z axis. The upper disk moves towards the lower disk normally with uniform velocity \dot{h}_0 .

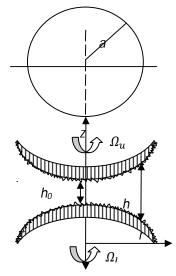


Fig 1. Configuration of the bearing system

The bearing surfaces are assumed to be transversely rough. Following the stochastic modeling of roughness employed in Christensen and Tonder [19, 20, 21] the film thickness h(x) of the lubricant film is taken as,

$$h(x) = \bar{h}(x) + h_s \tag{1}$$

where $\bar{h}(x)$ is the mean film thickness and h_s is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. The deviation h_s is governed by the probability density function

$$f(h_{s}) = \begin{cases} \frac{35}{32c^{7}} (c^{2} - h_{s}^{2})^{3}, -c \leq h_{s} \leq c \\ 0, & \text{elsewhere} \end{cases}$$

where c is the maximum deviation from the mean film thickness. The details regarding the mean α , the standard deviation σ and the parameter ϵ which is the measure of symmetry of the random variable h_s , are considered from the study of Christensen and Tonder [19, 20, 21].

It is assumed that the rotating upper plate lying along the surface determined by the relation

$$z_u = h_0 \exp(-\beta r^2); \ 0 \le r \le a$$

approaches with normal velocity $\dot{h_0}$ to the rotating lower plate lying along the surface given by

$$z_1 = h_0[exp(-\gamma r^2) - 1]; 0 \le r \le a$$

where β and γ are the curvature parameters of the corresponding plates and h_0 is the central film thickness. The film thickness h(r) then is defined by (Bhat [22], Abhangi and Deheri [16])

$$h(r) = h_0[exp(-\beta r^2) - exp(-\gamma r^2) + 1]; 0 \le r \le a$$

In 1967, Shliomis [23] pointed out that magnetic particles of a magnetic fluid can relax in two ways when the applied magnetic field changes. The first is by the rotation of magnetic particles in the fluid and the second by rotation of the magnetic moment with in the particles. The particle rotation is given by Brownian relaxation time parameters τ_B while the intrinsic rotational process is described by the relaxation time parameters τ_S . Assuming steady flow, neglecting inertial and second derivatives of \overline{S} , the equations governing the flow become,

$$-\nabla_{\rm p} + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M}.\nabla) \bar{H} + \frac{1}{2\tau_{\rm S}} \nabla \times (\bar{S} - I\bar{\Omega}) = 0$$
⁽²⁾

$$\overline{S} = I\overline{\Omega} + \mu_0 \tau_S(\overline{M} \times \overline{H})$$
(3)

$$\overline{M} = M_0 \frac{H}{H} + \frac{\tau_B}{I} (\overline{S} \times \overline{M})$$
(4)

where \overline{S} is the internal angular momentum, I is the sum of moments of inertia of the particles per unit volume, $\overline{\Omega} = \frac{1}{2}\nabla \times \overline{q}$, together with

$$\nabla \cdot \overline{\mathbf{q}} = \mathbf{0}, \nabla \times \overline{\mathbf{H}} = \mathbf{0}, \nabla \cdot (\overline{\mathbf{H}} + \overline{\mathbf{M}}) = \mathbf{0}$$

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(Bhat[22]), \bar{q} is the fluid viscosity in the film region, \bar{H} is external magnetic field, $\bar{\mu}$ is magnetic susceptibility of the magnetic field, p is the film pressure, η is the fluid viscosity and μ_0 is the permeability of the free space.

By making use of equation (3) in equation (2) and (4), one finds that,

$$-\nabla_{\mathrm{p}} + \eta \nabla^{2} \bar{\mathrm{q}} + \mu_{0} (\bar{\mathrm{M}} \cdot \nabla) \bar{\mathrm{H}} + \frac{1}{2} \mu_{0} \nabla \times (\bar{\mathrm{M}} \times \bar{\mathrm{H}}) = 0$$
(5)

and

$$\overline{M} = M_0 \frac{\overline{H}}{H} + \tau_B (\overline{\Omega} \times \overline{M})$$
(6)

Neglecting $\tau_B\,\tau_S$ terms, substitution of $\overline{\mathsf{M}}$ in above equation, leads to

$$\nabla_{p} + \left(\eta + \frac{\mu_{0}}{4}\tau_{B}\overline{M}.\overline{H}\right)\nabla^{2}\overline{q} + \mu_{0}(\overline{M}.\nabla)\overline{H} + \frac{1}{2}\mu_{0}\tau_{B}[\nabla(\overline{\Omega}.\overline{H}) \times \overline{M} + (\overline{\Omega}.\overline{H}).\nabla \times \overline{M} - \nabla(\overline{M}.\overline{H}) \times \overline{\Omega}] = 0$$
(7)

From equation (6), it is easily observed that an initial approximation to \overline{M} is $\overline{M} = M_0 \frac{\overline{H}}{\overline{H}}$

Substituting the value of \overline{M} on the right side of equation (6), a second approximation to \overline{M} is found to be

$$\overline{\mathsf{M}} = \mathsf{M}_0 \frac{\overline{\mathsf{H}}}{\mathsf{H}} + \frac{\mathsf{M}_0}{\mathsf{H}} \tau_{\mathrm{B}} (\overline{\Omega} \times \overline{\mathsf{H}})$$

Again, substituting this value of \overline{M} on the right side of equation (6), third approximation to \overline{M} is availed as

$$\overline{\mathsf{M}}.\,\overline{\mathsf{H}} = \mathsf{M}_0\mathsf{H} + \frac{\mathsf{M}_0}{\mathsf{H}}\tau_{\mathsf{B}}^2\big\{(\overline{\Omega}.\,\overline{\mathsf{H}})^2 - \Omega^2\mathsf{H}^2\big\} \tag{8}$$

In view of the equations of Shliomis model in cylindrical polar coordinates with uniform magnetic field, when both surfaces are solid and upper one rotates, the governing equation for the film pressure is obtained as [Bhat [22]],

$$\begin{aligned} \frac{1}{r}\frac{d}{dr}\left(h^{3}r\frac{dp}{dr}\right) &= 12\eta_{a}\dot{h_{0}} + \frac{3}{10}\rho\Omega_{u}^{2}\frac{1}{r}\frac{d}{dr}\left(r^{2}h^{3}\right) + \frac{3}{320}\frac{N\tau_{B}^{3}}{\eta_{a}^{3}}\frac{1}{r}\frac{d}{dr} \\ &\left[h^{5}r\left\{-\left(\frac{dp}{dr}\right)^{3} + \frac{27}{616}\rho^{3}r^{3}\Omega_{u}^{6} + \frac{13}{14}\left(\frac{dp}{dr}\right)^{2}\rho r\Omega_{u}^{2} - \frac{251}{756}\frac{dp}{dr}\rho^{2}r^{2}\Omega_{u}^{4}\right\}\right](9)\end{aligned}$$

Neglecting τ_B^3 term, following the discussions of Christensen and Tonder [19, 20, 21] regarding the modelling of roughness and under the usual assumptions of hydro-magnetic lubrication (Bhat [22], Prajapati [24], Deheri et al. [25]) the modified Reynolds equation when both plates rotate, takes the form

$$\frac{1}{r}\frac{d}{dr}\left(g(h)r\frac{dp}{dr}\right) = 12\eta(1+\tau)\dot{h_0} + \rho\left(\frac{3}{10}\Omega_r^2 + \Omega_r\Omega_l + \Omega_l^2\right)\frac{1}{r}\frac{d}{dr}(r^2g(h))$$
(10)

where

$$g(h) = h^3 + 3h^2\alpha + 3(\sigma^2 + \alpha^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon.$$

The following non dimensional quantities are adopted

$$\bar{\mathbf{n}} = \frac{\mathbf{h}}{\mathbf{h}_0}, \mathbf{R} = \frac{\mathbf{r}}{a}, \mathbf{P} = -\frac{\mathbf{h}_0^3 \mathbf{p}}{\eta a^2 \dot{\mathbf{h}}_0}, \mathbf{B} = \beta a^2, \mathbf{C} = \gamma a^2, \bar{\mathbf{\sigma}} = \frac{\mathbf{\sigma}}{\mathbf{h}_0}, \bar{\mathbf{\alpha}} = \frac{\alpha}{\mathbf{h}_0}, \bar{\mathbf{\varepsilon}} = \frac{\epsilon}{\mathbf{h}_0^3},$$
$$\eta_a = \eta (1 + \tau), \Omega_r = \Omega_u - \Omega_l, \mathbf{S} = -\frac{\rho \Omega_u^2 \mathbf{h}_0^3}{\eta \dot{\mathbf{h}}_0}, \Omega_f = \frac{\Omega_l}{\Omega_u}$$

The related boundary conditions are

$$P(1) = 0, \left(\frac{dP}{dR}\right)_{R=0} = 0$$
(11)

Solving equation (10) in view of boundary conditions (11) the expression for non dimensional fluid film pressure is found to be

$$P = -6(1 + \tau) \int_{1}^{R} \frac{R}{g(\bar{h})} dR - \frac{S}{20} (3\Omega_{f}^{2} + 4\Omega_{f} + 3)(1 - R^{2})$$
(12)

where

$$g(\overline{h}) = \overline{h}^3 + 3\overline{h}^2\overline{\alpha} + 3(\overline{\sigma}^2 + \overline{\alpha}^2)\overline{h} + 3\overline{\sigma}^2\overline{\alpha} + \overline{\alpha}^3 + \overline{\epsilon}$$

The load carrying capacity of the bearing system then, is determined by

$$W = -\frac{h_0^3}{2\pi\eta a^4 \dot{h_0}} W = \int_0^1 RP dR$$
(13)

which assumes the form

$$W = 3(1 + \tau) \int_{0}^{1} \frac{R^{3}}{g(\bar{h})} dR - \frac{S}{80} (3\Omega_{f}^{2} + 4\Omega_{f} + 3)$$
(14)

using the non dimensional quantities. *1.2. 3. Results and discussion*

It is seen that the dimensionless pressure distribution is calculated from equation (12) while equation (14) determines the non dimensional load carrying capacity for the bearing system. It is manifest that the non dimensional pressure distribution increases by

$$6\tau \int_{R}^{1} \frac{R}{g(\bar{h})} dR$$

while the increase in the load carrying capacity turns out to be

$$3 \tau \int_0^1 \frac{R^3}{g(\bar{h})} dR$$

as compared to the case of traditional lubricants.

From equation (14) it can be concluded that the bearing can support certain amount of load even when there is no flow. Setting the roughness parameters to be zero this discussion reduces to the performance of a Shliomis model based squeeze film between curved rotating circular plates. Further, considering the magnetization parameter to be zero one gets the performance of a squeeze film in curved rotating circular plates (Bhat [22]). Lastly, in the absence of curvature, this is the effect of rotation on squeeze film performance as analyzed in Wu [26].

It is noticed that the expression for non dimensional load carrying capacity is linear with respect to the magnetization parameter τ . Accordingly increasing values of τ result in increased load carrying capacity, as can be seen from Figures 2-7. It is clear that the load carrying capacity rises sharply with increasing magnetization.

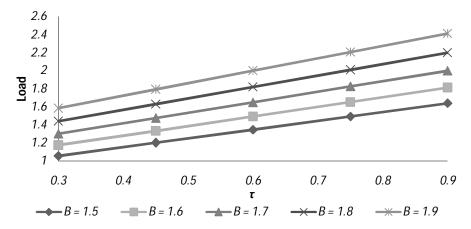


Fig 2. Variation of Load carrying capacity with respect to τ and *B*.

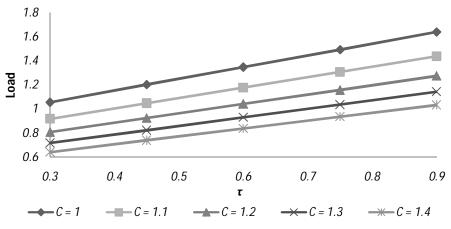
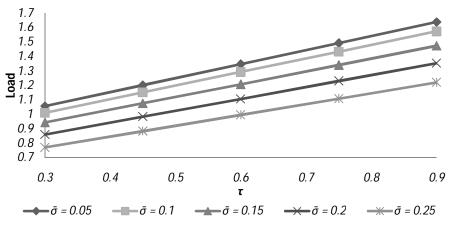
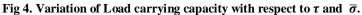


Fig 3. Variation of Load carrying capacity with respect to τ and *C*.





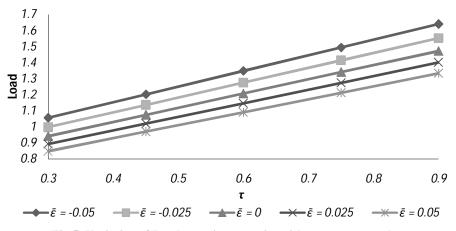


Fig 5. Variation of Load carrying capacity with respect to τ and $\overline{\epsilon}$.

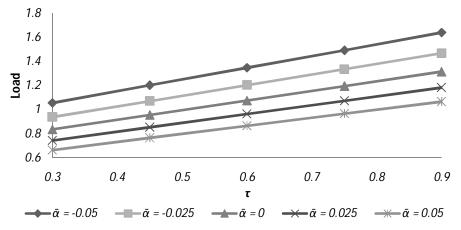
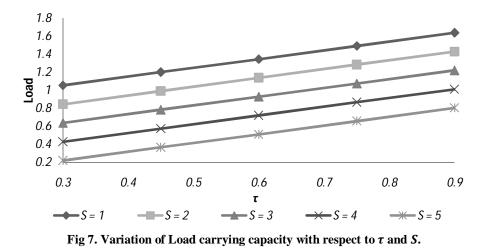


Fig 6. Variation of Load carrying capacity with respect to τ and $\overline{\alpha}$.



The effect of upper plate's curvature parameter is presented in Figures 8-12 while the influence of the lower plate's curvature parameter is shown in Figure 13-16. One can easily see that the effects of B and C are opposite in nature, in the sense that load carrying capacity increases considerably with the increase in the upper plate curvature parameter while increase in lower plate's curvature parameter results in decreased load carrying capacity. Thus, these graphical representations make it clear that for an effective performance the curvature parameters are required to be chosen appropriately.

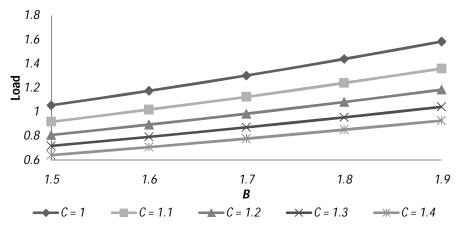


Fig 8. Variation of Load carrying capacity with respect to *B* and *C*.

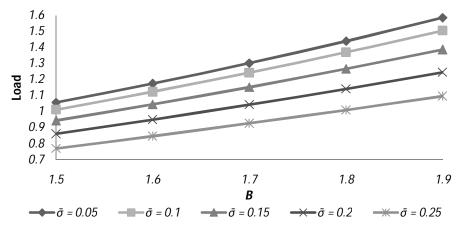


Fig 9. Variation of Load carrying capacity with respect to *B* and $\overline{\sigma}$.

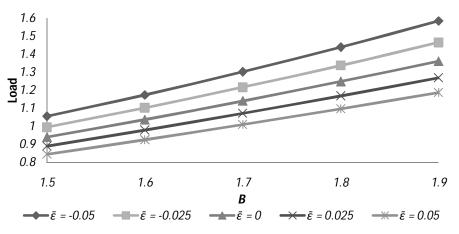


Fig 10. Variation of Load carrying capacity with respect to *B* and $\overline{\epsilon}$.

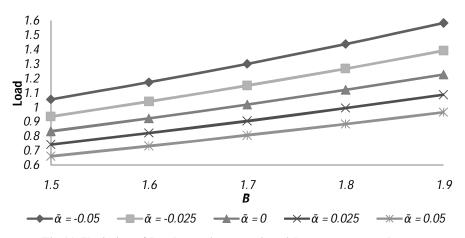


Fig 11. Variation of Load carrying capacity with respect to *B* and $\overline{\alpha}$.

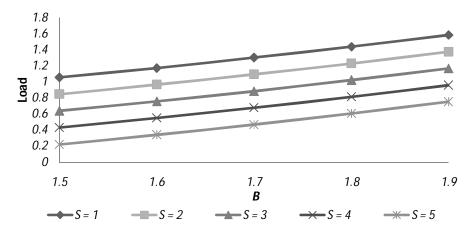
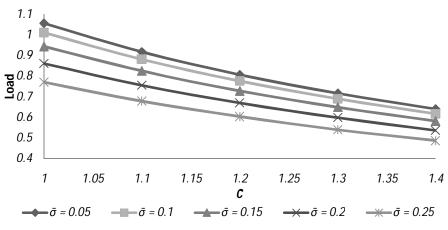
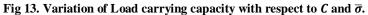


Fig 12. Variation of Load carrying capacity with respect to B and S.





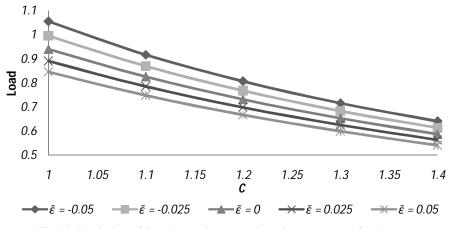
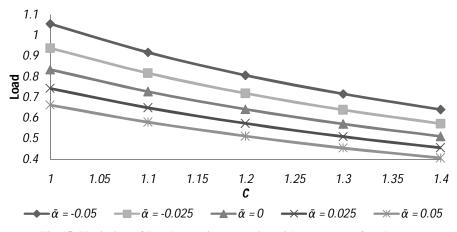
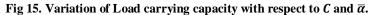
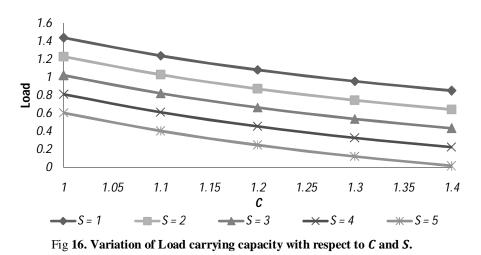


Fig 14. Variation of Load carrying capacity with respect to C and $\overline{\epsilon}$.







The effect of standard deviation on the distribution of load carrying capacity is considered adverse, as can be observed from Figures 17-19. The effect of skewness shown in Figures 20-21 establishes that positively skewed roughness decreases the load carrying capacity while the load carrying capacity gets raised owing to negatively skewed roughness.

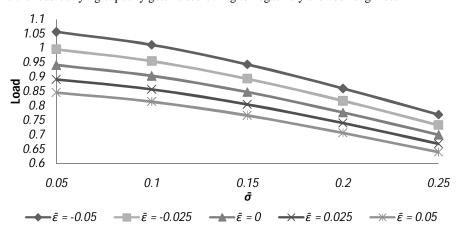


Fig 17. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and $\overline{\epsilon}$.

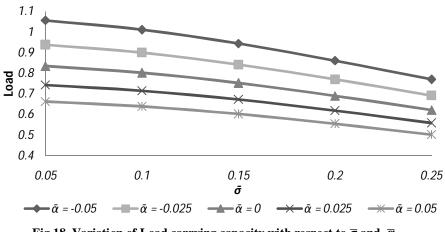


Fig 18. Variation of Load carrying capacity with respect to $\overline{\sigma}$ and $\overline{\alpha}$.

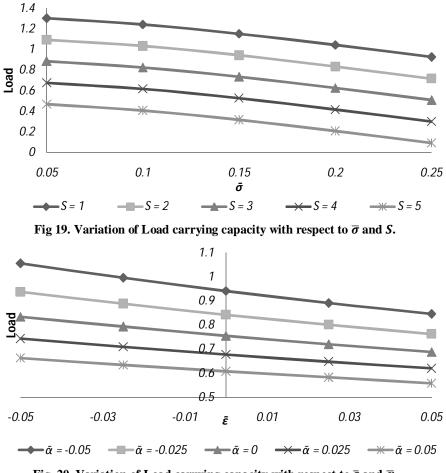


Fig. 20. Variation of Load carrying capacity with respect to $\overline{\epsilon}$ and $\overline{\alpha}$.

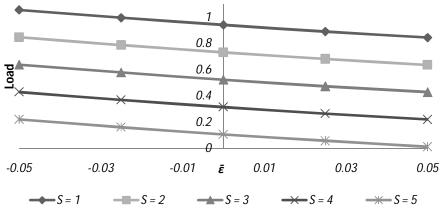


Fig 21. Variation of Load carrying capacity with respect to $\bar{\varepsilon}$ and *S*.

The effect of variance is observed to be quite similar to that of the trends of skewness which can be seen from Figure 22. Therefore, the combined effect of variance (-ve) and negatively skewed roughness is considerably positive.

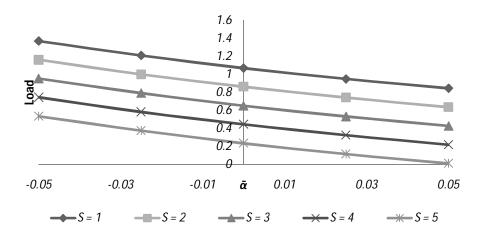


Fig 22. Variation of Load carrying capacity with respect to $\overline{\alpha}$ and *S*.

Lastly, the message from some of the Figures is that the effect of rotational inertia results in decreased load carrying capacity. Although, the effect of transverse surface roughness is adverse in general, this investigation suggests that this adverse effect of roughness can be minimized by the positive effect of magnetization suitably choosing the curvature parameters. The combined adverse effect of standard deviation and rotation can be compensated up to a large extent by the combined effect of magnetization and negatively skewed roughness especially when variance(-ve) is in place. *1.3.*

4. Conclusion

This investigation reveals that the Shliomis model is more suitable for this type of bearing system as compared to the Neuringer-Rosensweig model. The results presented in graphical forms indicate that the roughness must be accorded priority while designing the bearing system. Lastly, it is observed that this type of bearing system supports certain amount of load even in the absence of flow with an appropriate choice of rotational parameters which is very unlikely in the case of conventional lubricants.

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