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ON δ_p^α- CLOSED SETS, δ_p^α-CONTINUITY AND α-δ_p-ALMOST COMPACTNESS FOR CRISP SUBSETS IN A FUZZY TOPOLOGICAL SPACE

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ABSTRACT

This paper is a continuation of the study of α-δ_p-almost compactness for crisp subsets in a fuzzy topological space, initiated in [2] by using α-shading of Gantner et. al [6]. Here we introduce and study a new class of ordinary subsets, called δ_p^α-closed sets, which inherit α-δ_p-almost compactness of a space X (endowed with a fuzzy topology). Again a new type of function, called δ_p^α-continuous function, between two fuzzy topological spaces is introduced and shown that α-δ_p-almost compactness of crisp subsets remains invariant under this function.

Keywords:

α-shading, α-δ_p-almost compact set, δ_p^α-closed set, δ_p^α-continuity

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INTRODUCTION

The concept of α-shading in a fuzzy topological space was introduced by Gantner et al. [6]. This paved a new idea for generalizing different types of compactness to fuzzy perspective. Using this idea, in [2] we have introduced and studied α-δ_p-almost compactness for crisp subsets (i.e., an ordinary subset) in a fuzzy topological space.

Here in this paper a class of new concepts has been introduced with the related study. A class of crisp subsets, called δ_p^α-closed subsets, is introduced and established that α-δ_p- almost compactness of a space is inherited by such subsets. We have obtained a necessary condition for α-δ_p-almost compactness of crisp subsets via such subsets. Finally, we define two new types of functions under which α-δ_p-almost compactness and δ_p^α-closeness remain invariant respectively.

Throughout the paper, by an fts X, we denote a fuzzy topological space (X, τ) in the sense of Chang [4]. By a crisp subset of an fts X, we mean an ordinary subset A of X, i.e., $A \subseteq X$, where the underlying structure on X is a fuzzy topology whereas a fuzzy set A in an fts X denotes, as usual, a function from X to the closed interval $I = [0, 1]$, of the real line, i.e., $A \in I^X$ [8]. For a fuzzy set A in an fts X, clA and $intA$ will respectively stand for the fuzzy closure and interior of A in (X, τ) [7]. The support of a fuzzy set A in X will be denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point [7] in X with the singleton support $\{x\} \subseteq X$ and the value α ($0 < \alpha \leq 1$) will be denoted by x_α. For two fuzzy sets A and B in X, we write $A \leq B$ if $A(x) \leq B(x)$, for all $x \in X$ while we write AqB if A is quasi coincident (q-coincident, for short) with B [7], i.e., if there exists $x \in X$ such that $A(x) + B(x) > 1$; the negation of these two statements are written as $A \not\leq B$ and $A\bar{q}B$ respectively. A fuzzy set B is called a quasi-neighbourhood (q-nbd, for short) of a fuzzy set A if there is a fuzzy open set U in X such that $AqU \leq B$. If, in addition, B is fuzzy open, then B is called a fuzzy open q-nbd of A. A fuzzy nbd [7] A of a fuzzy point x_α in an fts X is define in the usual way, i.e., whenever for some fuzzy open set U in X, $x_\alpha \leq U \leq A$; A is a fuzzy open nbd of x_α if A is fuzzy open, in addition. A fuzzy set A in X is called a fuzzy regular open set if $int\, clA = A$[1]. A fuzzy point x_α is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nbd U of x_α is q-coincident with A [5]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A and is denote by δclA.

1. SOME WELL KNOWN DEFINITIONS

Let us now recall some definitions from [3] for ready references.

DEFINITION 1.1. A fuzzy set A in an fts X is said to be fuzzy δ-preopen if $A \leq int(\delta clA)$.

The complement of a fuzzy δ-preopen set is called fuzzy δ-preclosed.

DEFINITION 1.2. A fuzzy set A in an fts X is called a fuzzy δ-pre-q-nbd of a fuzzy point x_α in X if there exists a fuzzy δ-preopen set V in X such that $x_\alpha qV \leq A$.

DEFINITION 1.3. A fuzzy point x_α in an fts X is called a fuzzy δ-precluster point of a fuzzy set A in X if every fuzzy δ-pre-q-nbd of x_α is q-coincident with A.

The union of all fuzzy δ-precluster points of A is called the fuzzy δ-preclosure of A and will be denoted by $\delta - pclA$.

Next we recall the definition of α-shading from [6].

DEFINITION 1.4. Let X be an fts, and A be a crisp subset of X. A collection U of fuzzy sets in X is called an α-shading (where 0 <α<1) of A if for each $x \in A$, there is $U_x \in$ U such that $U_x(x) > \alpha$. Taking $A = X$, we arrive at the definition of α-shading of an fts X, as proposed by Gantner et.al [6]. If the members of an α-shading U of A (or of X) are fuzzy δ-preopen sets in X, then U is called a fuzzy δ-preopen α-shading of A (resp. of X).

DEFINITION 1.5 [2]. Let X be an fts and A, a crisp subset of X. A is said to be α-δ_p-almost compact if each fuzzy δ-preopen α-shading U of A has a finite δ_p-proximate α-subshading, i.e., there exists a finite subcollection U$_0$ of U such that $\{\delta - pclU : U \in$ U$_0\}$ is again an α-shading of A. If, in particular, $A = X$, then X is called an α-δ_p-almost compact space.

2. δ_p^α-CLOSED SET AND ITS APPLICATIONS

In this section, we introduce, as follows, a class of crisp sets in an fts.

DEFINITION 2.1. Let (X,τ) be an fts and $A \subseteq X$. A point $x \in X$ is said to be a δ_p^α-limit point of A if for every fuzzy δ-preopen set U in X with $U(x) > \alpha$, there exists $y \in A \setminus \{x\}$ such that $(\delta - pclU)(y) > \alpha$. The set of all δ_p^α-limit points of A will be denoted by $[A]_{\delta_p}^\alpha$.

The δ_p^α-closure of A, to be denoted by $\delta_p^\alpha - clA$, is defined by $\delta_p^\alpha - clA = A \cup [A]_{\delta_p}^\alpha$.

DEFINITION 2.2. A crisp subset A of an fts X is said to be δ_p^α-closed if it contains all its δ_p^α-limit points. Any subset B of X is called δ_p^α-open if $X \setminus B$ is δ_p^α-closed.

REMARK 2.3. For any $A \subseteq X$, it is clear that $A \subseteq \delta_p^\alpha - clA$, and $\delta_p^\alpha - clA = A$ iff $[A]_{\delta_p}^\alpha \subseteq A$. Then in view of Definition 2.1, it follows that A is δ_p^α-closed iff $\delta_p^\alpha - clA = A$. It is also clear that $A \subseteq B \subseteq X \Rightarrow [A]_{\delta_p}^\alpha \subseteq [B]_{\delta_p}^\alpha$.

THEOREM 2.4. A δ_p^α-closed subset A of an α-δ_p-almost compact space X is α-δ_p-almost compact.

PROOF. Let A ($\subseteq X$) be δ_p^α-closed in X. Then for any $x \notin A$, there is a fuzzy δ-preopen set U_x in X such that $U_x(x) > \alpha$, and $(\delta - pclU_x)(y) \leq \alpha$, for every $y \in A$. Consider the collection U = $\{U_x : x \notin A\}$. Now to prove that A is α-δ_p-almost compact, consider a fuzzy δ-preopen α-shading V of A. Clearly U$\cup$V is a fuzzy δ-preopen α-shading of X. Since X is α-δ_p-almost compact, there exists a finite subcollection $\{V_1, V_2, \ldots, V_n\}$ of U$\cup$V such that for every $t \in X$, there exists V_i ($1 \leq i \leq n$) such that $(\delta - pclV_i)(t) > \alpha$. For every member U_x of U , $(\delta - pclU_x)(y) \leq \alpha$, for every $y \in A$. So if this subcollection contains any member of U , we omit it and hence we get the result.

To achieve the converse of Theorem 2.4, we define the following :

DEFINITION 2.5. An fts (X,τ) is said to be α-δ_p-Urysohn if for any two distinct points x, y of X, there exists a fuzzy open set U and a fuzzy δ-preopen set V in X with $U(x) > \alpha$, $V(y) > \alpha$ and $\min((\delta - pclU)(z), (\delta - pclV)(z)) \leq \alpha$, for each $z \in X$.

THEOREM 2.6. An α-δ_p-almost compact set in an α-δ_p-Urysohn space X is δ_p^α-closed.

PROOF. Let A be α-δ_p-almost compact and $x \in X \setminus A$. Then for each $y \in A, x \neq y$. By α-δ_p-Urysohn property of X, there exist a fuzzy open set U_y and a fuzzy δ-preopen set V_y in X such that $U_y(x) > \alpha$, $V_y(y) > \alpha$ and $\min\left((\delta - pclU_y)(z), (\delta - pclV_y)(z)\right) \leq \alpha$, for all $z \in X$ … (1).

Then U = $\{V_y : y \in A\}$ is a fuzzy δ-preopen α-shading of A and so by α-δ_p-almost compactness of A, there are finitely many points $y_1, y_2, \ldots, y_n$ in A such that U$_0$ = $\{\delta - pclV_{y_1}, \ldots, \delta - pclV_{y_n}\}$ is again an α-shading of A. Now $U = U_{y_1} \cap \ldots \cap U_{y_n}$ being a fuzzy open set is a fuzzy δ-preopen set in X such that $U(x) > \alpha$. In order to show A to be δ_p^α-closed, it now suffices to show that $(\delta - pclU)(y) \leq \alpha$, for each $y \in A$. In fact, if for some $z \in A$, we assume $(\delta - pclU)(z) > \alpha$, then as $z \in A$, we have $(\delta - pclV_{y_k})(z) > \alpha$, for some k $(1 \leq k \leq n)$. Also $(\delta - pclU_{y_k})(z) > \alpha$. Hence $\min\left((\delta - pclU_{y_k})(z), (\delta - pclV_{y_k})(z)\right) > \alpha$, contradicting (1).

COROLLARY 2.7. In an α-δ_p-almost compact, α-δ_p-Urysohn space X, a subset A of X is α-δ_p-almost compact iff it is δ_p^α-closed.

THEOREM 2.8. In an α-δ_p-almost compact space X, every cover of X by δ_p^α-open sets has a finite subcover.

PROOF. Let U = $\{U_i : i \in \Lambda\}$ be a cover of X by δ_p^α-open sets. Then for each $x \in X$, there exists $U_x \in$ U such that $x \in U_x$. Now, as $X \setminus U_x$ is δ_p^α-closed, there exists a fuzzy δ-preopen set V_x in X such that $V_x(x) > \alpha$, and $(\delta - pclV_x)(y) \leq \alpha$, for each $y \in X \setminus U_x$ … (1). Then $\{V_x : x \in X\}$ forms a fuzzy δ-preopen α-shading of the α-δ_p-almost compact space X. Thus there exists a finite subset $\{x_1, x_2, \ldots, x_n\}$ of X such that $\{\delta - pclV_{x_i} : i = 1, 2, \ldots, n\}$ is an α-shading of X … (2).

We claim that $\{U_{x_1}, \ldots, U_{x_n}\}$ is a finite subcover of U. If not, then there exists $y \in X \setminus \bigcup_{i=1}^n U_{x_i}$ = $\bigcap_{i=1}^n (X \setminus U_{x_i})$. Then by (1), $(\delta - pclV_{x_i})(y) \leq \alpha$ for $i = 1, 2, \ldots, n$ and so $(\bigcup_{i=1}^n \delta - pclV_{x_i})(y) \leq \alpha$, contradicting (2).

THEOREM 2.9. Let (X, τ) be an fts. If X is α-δ_p-almost compact, then every collection of δ_p^α-closed sets in X with finite intersection property has non empty intersection.

PROOF. Let F = $\{F_i : i \in \Lambda\}$ be a collection of δ_p^α-closed sets in an α-δ_p-almost compact space X having finite intersection property. If possible, let $\bigcap_{i\in\Lambda} F_i = \emptyset$. Then $X \setminus \bigcap_{i\in\Lambda} F_i = \bigcup_{i\in\Lambda}(X \setminus F_i) = X \Rightarrow$ U = $\{X \setminus F_i : i \in \Lambda\}$ is an δ_p^α-open cover of X. Then by Theorem 2.8, there is a finite subset Λ_0 of Λ such that $\bigcup_{i\in\Lambda_0}(X \setminus F_i) = X \Rightarrow \bigcap_{i\in\Lambda_0} F_i = \phi$, a contradiction.

3. δ_p^α-CONTINUITY AND ITS APPLICATIONS

In this section we finally introduce a class of functions under which α-δ_p-almost compactness remains invariant.

DEFINITION 3.1. Let X, Y be fts's. A function $f : X \rightarrow Y$ is said to be δ_p^α-continuous if for each point $x \in X$ and each fuzzy δ-preopen set V in Y with $V(f(x)) > \alpha$, there is a fuzzy δ-preopen set U in X with $U(x) > \alpha$ such that $\delta - pclU \leq f^{-1}(\delta - pclV)$.

THEOREM 3.2. If $f : X \rightarrow Y$ is fuzzy δ_p^α-continuous (where X, Y are, as usual, fts's), then the following are true :

(a) $f\left([A]_{\delta_p}^\alpha\right) \subseteq [f(A)]_{\delta_p}^\alpha$, for every $A \subseteq X$.

(b) $[f^{-1}(A)]_{\delta_p}^\alpha \subseteq f^{-1}([A]_{\delta_p}^\alpha)$, for every $A \subseteq Y$.

(c) For each δ_p^α-closed set A in Y, $f^{-1}(A)$ is δ_p^α-closed in X.

(d) For each δ_p^α-open set A in Y, $f^{-1}(A)$ is δ_p^α-open in X.

PROOF.(a) Let $x \in [A]_{\delta_p}^\alpha$ and U be any fuzzy δ-preopen set in Y with $U(f(x)) > \alpha$. Then there is a fuzzy δ-preopen set V in X with $V(x) > \alpha$ and $\delta - pclV \leq f^{-1}(\delta - pclU)$. Now $x \in [A]_{\delta_p}^\alpha$ and V is a fuzzy δ -preopen set in X with $V(x) > \alpha \Rightarrow \delta - pclV(x_0) > \alpha$ for some $x_0 \in A \setminus \{x_0\} \Rightarrow \alpha < \delta - pclV(x_0) \leq (f^{-1}(\delta - pclU))(x_0) = (\delta - pclU)f(x_0)$ where $f(x_0) \in f(A) \setminus [f(x)] \Rightarrow f(x) \in [f(A)]_{\delta_p}^\alpha$. Thus (a) follows.

(b) By (a), $f\left([f^{-1}(A)]_{\delta_p}^\alpha\right) \subseteq [ff^{-1}(A)]_{\delta_p}^\alpha \subseteq [A]_{\delta_p}^\alpha \Rightarrow [f^{-1}(A)]_{\delta_p}^\alpha \subseteq f^{-1}([A]_{\delta_p}^\alpha)$.

(c) We have $[A]_{\delta_p}^\alpha = A$. By (b), $[f^{-1}(A)]_{\delta_p}^\alpha \subseteq f^{-1}\left([A]_{\delta_p}^\alpha\right) = f^{-1}(A) \Rightarrow [f^{-1}(A)]_{\delta_p}^\alpha = f^{-1}(A) \Rightarrow f^{-1}(A)$ is δ_p^α-closed in X.

(d) Follows from (c).

THEOREM 3.3. Let X, Y be fts's and $f : X \rightarrow Y$ be fuzzy δ_p^α-continuous. If $A\ (\subseteq X)$ is α-δ_p-almost compact, then so is $f(A)$ in Y.

PROOF. Let V = $\{V_i : i \in \Lambda\}$ be a fuzzy δ-preopen α-shading of $f(A)$, where A is α-δ_p-almost compact set in X. For each $x \in A$, $f(x) \in f(A)$ and so there exists $V_x \in$V such that $V_x(f(x)) > \alpha$. By fuzzy δ_p^α -continuity of f, there exists a fuzzy δ -preopen set U_x in X such that $U_x(x) > \alpha$ and $f(\delta - pclU_x) \leq \delta - pclV_x$. Then $\{U_x : x \in X\}$ is a fuzzy δ-preopen α-shading of A. By α-δ_p-almost compactness of A, there exist finitely many points $a_1, a_2, \ldots, a_n$ in A such that $\{\delta - pclU_{a_i} : i = 1, 2, \ldots, n\}$ is again an α-shading of A. We claim that $\{\delta - pclV_{a_i} : i = 1,, 2, \ldots, n\}$ is an α-shading of $f(A)$. In fact, $y \in f(A) \Rightarrow$ there exists $x \in A$ such that $y = f(x)$. Now there is a U_{a_j} (for some $j, 1 \leq j \leq n$) such that $\left(\delta - pclU_{a_j}\right)(x) > \alpha$ and hence $\left(\delta - pclV_{a_j}\right)(y) \geq f\left(\delta - pclU_{a_j}\right)(y) \geq \delta - pclU_{a_j}(x) > \alpha$.

We now define a function under which δ_p^α-closedness of a set remains invariant.

DEFINITION 3.4. Let X, Y be fts's. A function $f : X \to Y$ is said to be fuzzy δ-preopen if $f(A)$ is fuzzy δ-preopen in Y whenever A is fuzzy δ-preopen in X.

REMARK 3.5. For a fuzzy δ-preopen function $f : X \to Y$, every fuzzy δ-preclosed set A in X, $f(A)$ is fuzzy δ-preclosed in Y.

THEOREM 3.6. If $f : (X, \tau) \to (Y, \tau_1)$ is a bijective fuzzy δ-preopen function, then the image of a δ_p^α-closed set in (X, τ) is δ_p^α-closed in (Y, τ_1).

PROOF. Let $A\ (\subseteq X)$ be a δ_p^α-closed set in (X, τ) and let $y \in Y \setminus f(A)$. Then there exists a unique $x \in X$ such that $f(x) = y$. As $y \notin f(A), x \notin A$. Now, A being δ_p^α-closed in X, there exists a fuzzy δ-preopen set V in X such that $V(x) > \alpha$ and $\delta - pclV(p) \leq \alpha$, for each $p \in A$ … (1).

As f is fuzzy δ-preopen, $f(V)$ is a fuzzy δ-preopen set in Y, and also $(f(V))(y) = V(x) > \alpha$. Let $t \in f(A)$. Then there is a unique $t_0 \in A$ such that $f(t_0) = t$. As f is bijective and fuzzy δ-preopen, by Remark 3.5, $\delta - pclf(V) \leq f(\delta - pclV)$. Then $(\delta - pclf(V))(t) \leq f(\delta - pclV)(t) = \delta - pclV(t_0) \leq \alpha$, by (1). Thus y is not a δ_p^α-limit point of $f(A)$. Hence the result.

From Theorem 3.2(c) and Theorem 3.6, it follows that

COROLLARY 3.7. Let $f : X \to Y$ be a fuzzy δ_p^α-continuous, bijective, δ-preopen function. Then A is δ_p^α-closed in Y iff $f^{-1}(A)$ is δ_p^α-closed in X.

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