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GENERALIZATION OF ONE SIDED RIGHT NEIGHBOURS FOR BLOCK DESIGN OF DUAL DESIGN

ABSTRACT

This paper is concerned with neighbour design of dual design with parameters $v^*=s(s+1)$, $b^*=s^2$, $r^*=s$, $k^*=s+1$, $n_1^*=s^2$, $n_2^*= s-1$, $\lambda_1^*=1$, $\lambda_2^*=0$, ${}^{p1}{}_{11}^*=s(s-1)$, ${}^{p2}{}_{11}^*=s^2$, whether s is a prime number or power of a prime number. In this paper the one-sided right neighbours for every treatment upto s^{th} order are find out and it is observed that these neighbour treatments follow the property of circularity of the same order.

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1 INTRODUCTION

One of the general problems of experimental design is to avoid extraneous effects, which though not of interest to the experimenters, does contribute significantly to the variability in the experimental material. It is necessary to design the experiment so that variability arising from extraneous sources can be systematically controlled. If variability from extraneous sources is not controlled, experimental error will increase due to natural random causes, and additional variability from the attributable source. Recently, there has been a considerable interest in the use of some alternative methods of local control called nearest neighbour methods. Experiments in agriculture, horticulture, and forestry often show neighbour effects, that is, the response on a given plot is affected by the treatments on neighbouring plots as well as by the treatment applied to that plot. Adjacent plots of land should have about the same influence on these responses, whereas plots some distance apart likely have a different influence. When treatments are varieties, neighbour effects may be caused by differences in height, root vigour, or germination date, especially on small plots, such as are used in plant-breeding experiments. Neighbour designs were initially used in serology. Rees (1967) presented a technique used in virus research, which requires the arrangement in circles of samples from a number of virus preparations in such a way that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. He gave neighbour designs for every v up to 41 with $k \leq 10$ and $\lambda'=1$. Freeman (1979) defined some two-dimensional designs balanced for nearest neighbours and defined that it is sometimes important to arrange the treatments in field experimentation in such a way that at least one replicate of every treatment is very near to at least one replicate of all the other treatments. Wilkinson et al. (1983) defined a design to be partially neighbour- balanced if each treatment has each other treatment as a neighbours, on either side, at most once. The designs are called neighbour balanced at distance 2 by Azais et al. (1993), which have the property that for each ordered pair of distinct treatments there is exactly one plot that has the first chosen treatment as left neighbour and second chosen treatment as right neighbour. They also give a catalog of circular neighbour balanced designs with $t - 1$ blocks of size t or t blocks of size $t - 1$, where t is the number of treatments. Bailey (2003) has given some designs for studying one-sided neighbour effects including its merits and demerits. Laxmi and Rani (2009) obtained the patterns of neighbor treatments for every treatment of neighbour designs of the OS1 series considering two-sided (left and right) border plots. Laxmi and Parmita (2010) suggested a method of finding left-neighbours of a treatment in a neighbor design for OS2 series without constructing the actual design. Laxmi and Parmita (2011) further suggested a method of finding right neighbours for the OS2 series. Laxmi et al. (2013) further suggested a method of finding two-sided neighbours for the OS2 series and observe that these neighbours follows the property of circularity for a treatment.

2 Method of Constructing Dual Designs from the OS1 series:

Raghavarao, D. (1971) have shown that if D is any asymmetrical BIBD with parameters v, b, r, k, λ=1, then its dual D* is a two associate –class PBIBD with parameters, $v^*=b$, $b^*=v$, $r^*=k$, $k^*=r$, $n_1^*=k(r-1)$, $n_2^*= b-1-n_1^*$, $\lambda_1^*=1$, $\lambda_2^*=0$, ${}^{p1}{}_{11}^*=r-2+(k-1)^2$, , ${}^{p2}{}_{11}^*=k^2$. Given two designs D and D*, D* can be dual of D where the dual of a design is defined as a new design whose treatments and blocks are in correspondence with the blocks and treatments of the original design i.e. the number of treatments of d be equal to the number of blocks in D*, the number of blocks of D be equal to the number of treatments in D* , and the number of replications of D be equal to the number block sizes in D* and incidence is preserved (where a block and treatment are incident if the treatment is contained in the block and non-incident otherwise. Let $N = n_{ij}$ be the incidence matrix of order (v×b) of a BIB design with parameters, $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$, that is,

$$n_{ij} = 1 \text{ if } i^{th} \text{ treatment occur in } j^{th} \text{ block.}$$
$$= 0 \text{ otherwise.}$$

Then it is obvious that every row and column of N matrix contains 1 in r and k places respectively and that any two rows of N contain the pair (1,1) in exactly λ columns. Conversely, any (0,1) matrix having these properties can be regarded as the incidence matrix of a BIB design. Thus the incidence matrix has the form:

$$\begin{matrix} n_{11} & n_{12} & n_{13} & . & . & . & n_{1b} \\ n_{21} & n_{22} & n_{23} & . & . & . & n_{2b} \\ n_{31} & n_{32} & n_{33} & . & . & . & n_{3b} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ n_{vb} & n_{vb} & n_{vb} & . & . & . & n_{vb} \end{matrix}$$

If N is the incidence matrix of the above BIB design, then by interchanging rows in columns and columns in rows or the transpose of the incidence matrix denoted by N' is the incidence matrix of the dual design.

3 Method of Constructing Neighbour Designs from the Dual Design

In 1936 Yates introduced the concept of orthogonal series for BIB designs with parameters ; $v=s^2$, $b=s(s+1)$,$r=s+1$,$k=s$,$\lambda=1$, and $v=b=s^2+s+1$, $r=k=s+1$, $\lambda=1$. The first series was named as OS1 series and second series was named as OS2 series. The OS1 series is asymmetrical BIBD and OS2 series is symmetrical BIB design. A design is said to be symmetrical iff v=b, that is, the number of treatments is equal to the number of treatments and asymmetrical iff $v\neq b$, that is ,the number of treatments is not equal to the number of treatments. This paper interests to the neighbour design of dual design constructed from the asymmetrical BIB design where parameters of the dual design (D*) is $v^*=s(s+1)$, $b^*=s^2$, $r^*=s$, $k^*=s+1$, $n_1^*=s^2$, $n_2^*= s-1$, $\lambda_1^*=1$, $\lambda_2^*=0$, ${}^{p1}{}_{11}^*=s(s-1)$, ${}^{p2}{}_{11}^*=s^2$.

3.1 Neighbour Design When s=3

Let us consider the case when s is the prime number. So the parameters of the design (D) thus becomes: v=9, b=12, r=4, k=3 and $\lambda=1$. For the construction of BIBD with these parameters one may refer to Laxmi and Rani (2009), which is as follows:

1	2	3
4	5	6
7	8	9
1	4	7
2	5	8
3	6	9
1	6	8
2	4	9
3	5	7
1	5	9
2	6	7
3	4	8

Let $N= (n_{ij})$ where i=1,…,v & j=1,…,b be the incidence matrix of order (v,b) i.e.

$n_{ij}=1$ if treatment i occur in block j.
=0 otherwise.

Thus the incidence matrix N of design D thus has the form

Blocks → / Treatments ↓	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0	0	1	0	0	1	0	0	1	0	0
2	1	0	0	0	1	0	0	1	0	0	0	1
3	1	0	0	0	0	1	0	0	1	0	1	0
4	0	1	0	1	0	0	0	1	0	0	1	0
5	0	1	0	0	1	0	0	0	1	1	0	0
6	0	1	0	0	0	1	1	0	0	0	0	1
7	0	0	1	1	0	0	0	0	1	0	0	1
8	0	0	1	0	1	0	1	0	0	0	1	0
9	0	0	1	0	0	1	0	1	0	1	0	0

The transpose of incidence matrix N is defined by N' which is as follows

blocks→ / treatments↓	1	2	3	4	5	6	7	8	9
1	1	1	1	0	0	0	0	0	0
2	0	0	0	1	1	1	0	0	0
3	0	0	0	0	0	0	1	1	1
4	1	0	0	1	0	0	1	0	0
5	0	1	0	0	1	0	0	1	0
6	0	0	1	0	0	1	0	0	1
7	1	0	0	0	0	1	0	1	0
8	0	1	0	1	0	0	0	0	1
9	0	0	1	0	1	0	1	0	0
10	1	0	0	0	1	0	0	0	1
11	0	0	1	1	0	0	0	1	0
12	0	1	0	0	0	1	1	0	0

With this incidence matrix the resulting design is the D_1* design with parameters v*=12, b*=9, r*=3, k*=4, n_1*=9, n_2*=2, λ_1*=1, λ_2*=0, $^{p1}{}_{11}$*=6, $^{p2}{}_{11}$*=9.

1	4	7	10
1	5	8	11
1	6	9	12
2	4	8	12
2	5	9	10
2	6	7	11
3	4	9	11
3	5	7	12
3	6	8	10

From the dual design D_1* the neighbour design is obtained using the method of border plots. The neighbour design thus obtained from D_1* can be written as

10	1	4	7	10	1
11	1	5	8	11	1
12	1	6	9	12	1
12	2	4	8	12	2
10	2	5	9	10	2
11	2	6	7	11	2
11	3	4	9	11	3
12	3	5	7	12	3
10	3	6	8	10	3

3.1.1 First-order neighbours of a treatment

From the above neighbour design we observed that in block 1 treatment number 1 has treatment number 4 as a first-order right neighbour. In block 2 treatment number 1 has treatment number 5 as a first-order right neighbour. Similarly, it can be easily be found the other neighbours of first-order for treatment number 1 in which block this treatment number appears. Thus a list of right neighbours of first-order for treatment number 1 is 4,5,6. In block 1 treatment number 4 has treatment number 7 as a first-order right neighbour and in block 4 treatment number 8 is the first-order right neighbour. Similarly, all the one-sided neighbours of first-order for treatment number 4 can be obtained in which block treatment number 4 appears. Thus a list of right neighbours of first-order for treatment number 4 is 7,8,9. So, the right neighbour treatments of first-order for every treatment can be found and table 1 is given for first-order neighbour treatments for every treatment is as follows:

Table 1

Treatments (i)	Series in which (i) lies	First-order common right neighbours	First-order common right neighbours series
1	$1 \le i \le s$	4,5,6	$s+1 \le i \le 2s$
2		4,5,6	
3		4,5,6	
4	$s+1 \le i \le 2s$	7,8,9	$2s+1 \le i \le s^2$
5		7,8,9	
6		7,8,9	
7	$2s+1 \le i \le s^2$	10,11,12	$s^2+1 \le i \le s^2+s$
8		10,11,12	
9		10,11,12	
10	$s^2+1 \le i \le s^2+s$	1,2,3	$1 \le i \le s$
11		1,2,3	
12		1,2,3	

3.1.2 Second-order neighbours of a treatment

From the neighbour design we observed that in block 1 treatment number 1 has treatment treatment number 7 as a second-order right neighbour. In block 2 treatment number 1 has treatment number 8 second-order right neighbour. Similarly, other neighbours of treatment 1 can easily be found from the blocks in which block this treatment number appear. Thus a list of one- dimensional neighbours of second-order for treatment number 1 as 7,8,9. In block 1 treatment number 4 has treatment number 10 as a second-order right neighbour. In block 4 treatment number 4 has treatment number 11 as the second-order right neighbour. Similarly, all the one-sided neighbours of second-order for treatment number 4 can be obtained in which block treatment number 4 appears. Thus a list of one- dimensional neighbours for treatment number 4 is 10,11,12. Thus, for every other treatment the right neighbour treatments of second-order can be found easily. Second-order neighbour treatments for every treatment is given in Table number 2.

Table 2

Treatments (i)	Series in which (i) lies	Second-order common right neighbours	Second-order common right neighbours series
1	$1 \le i \le s$	7,8,9	$2s+1 \le i \le s^2$
2		7,8,9	
3		7,8,9	
4	$s+1 \le i \le 2s$	10,11,12	$s^2+1 \le i \le s^2+s$
5		10,11,12	
6		10,11,12	
7	$2s+1 \le i \le s^2$	1,2,3	$1 \le i \le s$
8		1,2,3	
9		1,2,3	
10	$s^2+1 \le i \le s^2+s$	4,5,6	$s+1 \le i \le 2s$
11		4,5,6	
12		4,5,6	

3.1.3 Third-order neighbours of a treatment:

From the neighbour design we observed that in block 1 treatment number 1 has treatment number 10 as a third-order right neighbour. In block 2 treatment number 1 has treatment number 12 as a third-order right neighbour. Similarly, it can easily be found from the blocks in which this treatment number appears. Thus a list of one- dimensional neighbours of third-order for treatment number 1 is 10,12,11. Again in block 1 treatment number 4 has treatment number 1 as a third-order right neighbour. In block 4 treatment number 4 has treatment number 2 as the third-order right neighbour. Similarly, all the one-sided neighbours of third-order for treatment number 4 can be obtained in which block treatment number 4 appears. Thus a list of one- dimensional neighbours for treatment number 4 is 1,2,3. Thus, for every other treatment the right neighbour treatments of third-order can be found easily. Third-order neighbour treatments for every treatment is given in table number 3.

Table 3

Treatments (i)	Series in which (i) lies	Third-order common right neighbours	Third-order common right neighbours series
1	$1 \le i \le s$	10,11,12	$s^2+1 \le i \le s^2+s$
2		10,11,12	
3		10,11,12	
4	$s+1 \le i \le 2s$	1,2,3	$1 \le i \le s$
5		1,2,3	
6		1,2,3	
7	$2s+1 \le i \le s^2$	4,5,6	$1 \le i \le s$
8		4,5,6	
9		4,5,6	
10	$s^2+1 \le i \le s^2+s$	7,8,9	$2s+1 \le i \le s^2$
11		7,8,9	
12		7,8,9	

Now, we considered the fourth –ordered neighbour treatments for every treatment of the neighbour design and found that these fourth-ordered neighbours are the treatments themselves. Further higher-ordered neighbours ,i.e., fifth-ordered and so on, is the repetition of first-order, second-order and third-order neighbour treatments respectively. It implies that neighbours for a treatment upto third-order (s-th) can be obtained after that the repetition of treatments is there.

3.2 Neighbour design when s=4

Let us consider the case when s is the power of a prime number. So the parameters of the design (D) thus becomes: v=16,b=20,r=5,k=4 and λ=1. For the construction of BIBD with these parameters one may refer to Laxmi and Rani (2009). Dual design with the parameters of OS1 series can be easily constructed with the help of incidence matrix as discussed above. Parameters of Interested design D_2^* becomes $v^*=20$, $b^*=16$, $r^*=4$, $k^*=5$, $n_1^*=16$, $n_2^*=3$, $\lambda_1^*=1$, $\lambda_2^*=0$, $p_{11}^{1*}=12$, $p_{11}^{2*}=16$, D_2^* i.e. Dual design thus obtained is:

1	5	9	13	17
1	6	10	15	20
1	7	11	16	18
1	8	12	14	19
2	5	10	14	18
2	6	9	16	19
2	7	12	15	17
2	8	11	13	20
3	5	11	15	19
3	6	12	13	18
3	7	9	14	20
3	8	10	16	17
4	5	12	16	20
4	6	11	14	17

4	7	10	13	19
4	8	9	15	18

From the above design neighbour design can be obtained by using method of border plots and the neighbour design so obtained from D_2* is written as

17	1	5	9	13	17	1
20	1	6	10	15	20	1
18	1	7	11	16	18	1
19	1	8	12	14	19	1
18	2	5	10	14	18	2
19	2	6	9	16	19	2
17	2	7	12	15	17	2
20	2	8	11	13	20	2
19	3	5	11	15	19	3
18	3	6	12	13	18	3
20	3	7	9	14	20	3
17	3	8	10	16	17	3
20	4	5	12	16	20	4
17	4	6	11	14	17	4
19	4	7	10	13	19	4
18	4	8	9	15	18	4

3.2.1 First-order neighbours of a treatment

From the above neighbour design we observed that in block 1 treatment number 1 has treatment number 5 as a first-order right neighbour. In block 2 treatment number 1 has treatment number 6 as a first-order right neighbour. Similarly, it can be easily be found the other neighbours of first-order for treatment number 1 in which block this treatment number appears. Thus a list of right neighbours of first-order for treatment number 1 is 5,6,7,8. In block 1 treatment number 5 has treatment number 9 as a first-order right neighbour. In block 5 treatment number 10 is the first-order right neighbour. Similarly, all the one-sided neighbours of first-order for treatment number 5 can be obtained in which block treatment number 5 appears. Thus a list of right neighbours of first-order for treatment number 5 is 9,10,11,12. So, the right neighbour treatments of first-order for every treatment can be found easily and the resulted table number 4 is given for first-order neighbour treatments for every treatment is as follows:

Table 4

Treatments (i)	Series in which (i) lies	First-order common right neighbours	First-order common right neighbours series
1	$1 \le i \le s$	5,6,7,8	$s+1 \le i \le 2s$
2		5,6,7,8	
3		5,6,7,8	
4		5,6,7,8	
5	$s+1 \le i \le 2s$	9,10,11,12	$2s+1 \le i \le 3s$
6		9,10,11,12	
7		9,10,11,12	
8		9,10,11,12	
9	$2s+1 \le i \le 3s$	13,14,15,16	$3s+1 \le i \le s^2$
10		13,14,15,16	
11		13,14,15,16	

12		13,14,15,16	
13	$3s+1 \le i \le s^2$	17,18,19,20	$s^2+1 \le i \le s^2+s$
14		17,18,19,20	
15		17,18,19,20	
16		17,18,19,20	
17	$s^2+1 \le i \le s^2+s$	1,2,3,4	$1 \le i \le s$
18		1,2,3,4	
19		1,2,3,4	
20		1,2,3,4	

3.2.2 Second -order neighbours of a treatment

From the neighbour design we observed that in block 1 treatment number 1 has treatment number 9 as a second-order right neighbour. In block 2 treatment number 1 has treatment number 10 as a second-order right neighbour. Similarly, other neighbours of treatment 1 can easily be found from the blocks in which block this treatment number appears. Thus a list of one-dimensional neighbours of second-order for treatment number 1 as 9,10,11,12. In block 1 treatment number 5 has treatment number 13 as a second-order right neighbour. In block 5 treatment number 5 has treatment number 14 is the second-order right neighbour. Similarly, all the one-sided neighbours of second-order for treatment number 5 can be obtained in which block treatment number 5 appears. Thus a list of one- dimensional neighbours for treatment number 5 as 13,14,15,16. So, for every other treatment the right neighbour treatments of second-order can be found easily. The resulted table number 5 is given for second-order neighbour treatments for every treatment is as follows:

Table 5

Treatments (i)	Series in which (i) lies	Second-order common right neighbours	Second–order common right neighbour series
1	$1 \le i \le s$	9,10,11,12	$2s+1 \le i \le 3s$
2		9,10,11,12	
3		9,10,11,12	
4		9,10,11,12	
5	$s+1 \le i \le 2s$	13,14,15,16	$3s+1 \le i \le s^2$
6		13,14,15,16	
7		13,14,15,16	
8		13,14,15,16	
9	$2s+1 \le i \le 3s$	17,18,19,20	$s^2+1 \le i \le s^2+s$
10		17,18,19,20	
11		17,18,19,20	
12		17,18,19,20	
13	$3s+1 \le i \le s^2$	1,2,3,4	$1 \le i \le s$
14		1,2,3,4	
15		1,2,3,4	
16		1,2,3,4	
17	$s^2+1 \le i \le s^2+s$	5,6,7,8	$s+1 \le i \le 2s$
18		5,6,7,8	
19		5,6,7,8	
20		5,6,7,8	

3.2.3 Third-order neighbours of a treatment

From the above neighbour design we observed that in block 1 treatment number 1 has treatment number 13 as a third-order right neighbour. In block 2 treatment number 1 has treatment number 15 as a third-order right neighbour. Similarly, it can easily be found the other neighbours of treatment number 1 in which block this treatment number appears. Thus a list of one- dimensional neighbours of third-order for treatment number 1 as 13,15,16,14. In block 1 treatment number 5 has treatment number 17 as a third-order right neighbour. In block 5 treatment number 5 has treatment number 18 is the third-order right neighbour. Similarly, all the one-sided neighbours of third-order for treatment number 5 can be obtained in which block treatment number 5 appears . Thus a list of one- dimensional neighbours for treatment number 5 as 17,18,19,20. So, for every other treatment the right neighbour treatments of third-order can be found easily. The resulted table number 6 for third-order neighbour treatments for every treatment is as follows:

Table 6

Treatments (i)	Series in which (i) lies	Third-order common right neighbours	Third–order common right neighbour series
1	$1 \le i \le s$	13,14,15,16	$3s+1 \le i \le s^2$
2		13,14,15,16	
3		13,14,15,16	
4		13,14,15,16	
5	$s+1 \le i \le 2s$	17,18,19,20	$s^2+1 \le i \le s^2+s$
6		17,18,19,20	
7		17,18,19,20	
8		17,18,19,20	
9	$2s+1 \le i \le 3s$	1,2,3,4	$1 \le i \le s$
10		1,2,3,4	
11		1,2,3,4	
12		1,2,3,4	
13	$3s+1 \le i \le s^2$	5,6,7,8	$s+1 \le i \le 2s$
14		5,6,7,8	
15		5,6,7,8	
16		5,6,7,8	
17	$s^2+1 \le i \le s^2+s$	9,10,11,12	$2s+1 \le i \le 3s$
18		9,10,11,12	
19		9,10,11,12	
20		9,10,11,12	

3.2.4 Fourth-order neighbours of a treatment

From the above neighbour design we observed that in block 1 treatment number 17 as a fourth-order right neighbour. In block 2 treatment number 1 has treatment number 20 as a fourth-order right neighbour. Similarly, it can easily be found the other neighbours of treatment number 1 in which block this treatment number appears. Thus a list of one- dimensional neighbours of fourth-order for treatment number 1 as 17,20,18,19. In block 1 treatment number 5 has treatment number 1 as a fourth-order right neighbour. In block 5 treatment number 5 has treatment number 2 is the fourth-order right neighbour. Similarly, all the one-sided neighbours of fourth-order for treatment number 5 can be obtained in which block treatment number 5 appears . Thus a list of one- dimensional neighbours of fourth-order for treatment number 5 as 1,2,,3,4. So, for every other treatment the right neighbour treatments of fourth-order can be found easily. The resulted table number 7 for fourth-order neighbour treatments for every treatment is as follows:

Table 7

Treatments (i)	Series in which (i) lies	Fourth-order common right neighbours	Fourth –order common right neighbour series
1	$1 \le i \le s$	17,18,19,20	$s^2+1 \le i \le s^2+s$
2		17,18,19,20	
3		17,18,19,20	
4		17,18,19,20	
5	$s+1 \le i \le 2s$	1,2,3,4	$1 \le i \le s$
6		1,2,3,4	

7		1,2,3,4	
8		1,2,3,4	
9	$2s+1\le i\le 3s$	5,6,7,8	$s+1\le i\le 2s$
10		5,6,7,8	
11		5,6,7,8	
12		5,6,7,8	
13	$3s+1\le i\le s^2$	9,10,11,12	$2s+1\le i\le 3s$
14		9,10,11,12	
15		9,10,11,12	
16		9,10,11,12	
17	$s^2+1\le i\le s^2+s$	13,14,15,16	$3s+1\le i\le s^2$
18		13,14,15,16	
19		13,14,15,16	
20		13,14,15,16	

Now, we considered the fifth –ordered neighbour treatments for every treatment of the neighbour design and found that these fifth-ordered neighbours are the treatments themselves. Further higher-ordered neighbours ,i.e., sixth-ordered and so on, is the repetition of first-order, second-order and third-order ,fourth-order neighbour treatments respectively. It implies that neighbours for a treatment upto fourth-order (s-th) can be obtained after that the repetition of treatments is there.

4 Neighbours of Dual design for any value of s

The parameters of the neighbour design for Dual design are : $v^*=s(s+1)$, $b^*=s^2$, $r^*=s$, $k^*=s+1$, $n_1^*=s^2$, $n_2^*= s-1$, $\lambda_1^*=1$, $\lambda_2^*=0$, $p^1_{11}{}^*=s-1+(s-1)^2$, , $p^2_{11}{}^*=s^2$ for which one-sided neighbours are to be found whether s is either a prime number or power of a prime number.

(i) Consider the treatment number 'i' where $i=s^2+s$

(ii) Then find the series in which the treatment number 'i' lies

The series is defined in such a way that the sequence of first 's' treatments i.e. (1 to s) of the design form the first series, the sequence of next 's' treatments i.e. (s+1 to 2s) form the second series and the sequence of next 's' treatments i.e. (2s+1 to 3s) form the third series and so on so the last series consists of the extreme last treatment from (s^2+1)- th to (s^2+s)-th treatment. Thus, there are 's+1' series upto the treatment number s^2+s. The (s+2)-th series of treatment numbers s^2+s+1 to s^2+2s reduces to 1 to s with modv. So the (s+3)-rd series is again the first series of the design. It again holds true for the next (s+4)-th and so on series which proves that the design is circular.

(iii) Then find out the common right neighbour series for any treatment number:

Let the treatment number 'i' lies in the j^{th} series where (j=1,2,…,s) (j+1)-th series i.e. the next series is the first- order common right neighbour series and (j+2)-th series is the second-order right neighbour series. For example: let the treatment number 'i' lies in the j-th series where (j=1,…,s) then (j+2)-th series is the one series away in right direction is the second order common right neighbour series. In the similar fashion, r-th order right neighbour series can be observed where r=1,…,s. As there are s replications of each treatment there will be s total number of right neighbours of any order for a treatment.

Hence the systematic way of finding right neighbors of r-th order of any treatment is summarized in Table number 8

'i'	Series in which 'i' lies	1^{st}-order right nbhrs	2^{nd} order right nbhr series	. . .	s^{th} order right nbhr series
1 2 . . . S	$1 \le i \le s$	$s+1 \le i \le 2s$	$2s+1 \le i \le 3s$	. . .	$s^2+1 \le i \le s^2+s$
s+1 s+2 . . . 2s	$s+1 \le i \le 2s$	$2s+1 \le i \le 3s$	$3s+1 \le i \le 4s$	. . .	$s(s-1)+1 \le i \le s^2$
2s+1 2s+2 . . . 3s	$2s+1 \le i \le 3s$	$3s+1 \le i \le 4s$	$4s+1 \le i \le 5s$	. . .	$s^2+1 \le i \le s^2+s$
. . .	. . .	. . .	. . .	...	. . .
s(s-1)+1 s(s-1)+2 . . . s^2	$s(s-1)+1 \le i \le s^2$	$s^2+1 \le i \le s^2+s$	$1 \le i \le s$	. . .	$s(s-2)+1 \le i \le s(s-1)$
s^2+1 s^2+2 . . . s^2+s	$s^2+1 \le i \le s^2+s$	$1 \le i \le s$	$s+1 \le i \le 2s$	. . .	$s(s-1)+1 \le i \le s^2$

In the above table the first-order, second-order and so on upto (s)-th ordered neighbour treatment are given. After observing the (s+1)-th order one-sided neighbours it was found that these (s+1)-th ordered neighbours are the treatments themselves. Further, higher-ordered neighbours that is (s+2)-th, (s+3)-rd and so on is the repetition of first-order, second-order upto the s-th ordered neighbour treatments respectively. It implies that neighbours for a treatment upto s-th order (highest) can be obtained after that the repetition takes place.

5 CONCLUSION

In case of PBIBD series where $v^*=s(s+1)$, $b^*=s^2$, $r^*=s$, $k^*=s+1$, $n_1^*=s^2$, $n_2^*= s-1$, $\lambda_1^*=1$, $\lambda_2^*=0$, ${}^{p1}{}_{11}^*=s(s-1)$, ${}^{p2}{}_{11}^*=s^2$ where s is either a prime number or power of a prime number the circularity holds for the complete design in each set of s series. It is observed that for a given value of s, the neighbour treatments can be find out from 1-st order to (s-)-th order.

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