Carib.J.Sci.Tech

A study on vibrations in a composite poroelastic concentric cylinder

Authors & Affiliation:

J. Manoj Kumar[1], C. NageswaraNath[2,*], S. Ahmed Shah[3]

[1]Department of Mathematics, Hyderabad , India

[2]Department of Mathematics, CMR Technical Campus, Hyderabad - 501 401, India

[3]Department of Mathematics, Deccan college of Engineering and Technology, Hyderabad-500 001, India

KEY WORDS:
core, casing, rigidity, phase velocity, wave number, pervious surface.

ISSN 0799-3757

http://caribjscitech.com/

Abstract:

The problem of vibrations of an infinitely long poroelastic composite hollow cylinder is solved by employing Biot's theory of wave propagation in poroelastic media. A poroelastic composite hollow cylinder consists of two concentric poroelastic cylindrical layers both of which are made of different poroelastic materials with each poroelastic material as homogeneous and isotropic. The frequency equation of vibrations of poroelastic composite hollow cylinder is obtained along with some particular cases. Non-dimensional Phase velocity is computed as a function of wave number. The results are presented graphically for two types of poroelastic composite cylinders and then discussed. The vibrations of poroelastic composite hollow cylinder related to core and casing for pervious surface are uncoupled when the solid in casing is rigid.

1. Introduction

Gazis (1959) studied the propagation of free harmonic waves in elastic hollow circular cylinder. McNiven et al. (1963) discussed propagation of axially symmetric waves in composite elastic rods. Cui et al. (1997) and Abousleiman and Cui (1998) presented poroelastic solutions in an inclined borehole and transversely isotropic well-bore cylinders. Ahmed shah and Tajuddin (2009) discussed axially symmetric vibrations of finite composite poroelastic cylinders. Malla Reddy and Tajuddin (2010) studied axially symmetric vibrations of composite poroelastic cylinders. Flexural wave propagation in coated poroelastic cylinders is presented by Ahmed shah (2011). Tajuddin (2011) et al. discussed axial shear vibrations in poroelastic composite cylinder. Shanker et. al. (2012) studied radial vibrations in an infinitely long poroelastic coated cylinder.

In the present analysis, vibrations of an infinitely long poroelastic composite circular cylinder are investigated employing Biot's (1956) theory of wave propagation in porous materials. Biot's model consists of an elastic matrix permeated by a network of interconnected spaces saturated with liquid. The frequency equations of vibrations are obtained for poroelastic composite cylinder and as well for some particular cases i.e., poroelastic composite hollow cylinder with rigid casing, poroelastic composite bore and poroelastic bore. Non-dimensional phase velocity as a function of wavenumber is computed in each case i.e., poroelastic composite hollow cylinder, poroelastic composite bore and poroelastic composite cylinder with rigid casing. The results are presented graphically for two types of poroelastic composite cylinders and then discussed.

2. Basic equations, Formulation and solution of the Problem

The equations of motion of a homogeneous, isotropic poroelastic solid (Biot 1956) in the presence of dissipation *b* are:

$$N\nabla^2\mathbf{u}+(A+N)\nabla e+Q\nabla\varepsilon=\frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u}+\rho_{12}\mathbf{U})+b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U})$$

$$Q\nabla e+R\nabla\varepsilon=\frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u}+\rho_{22}\mathbf{U})-b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}) \qquad (1)$$

where ∇^2 is the Laplacian operator $\mathbf{u}$(u, v, w) and $\mathbf{U}$(U, V, W) are solid and liquid displacements ; e and ε are the dilatations of solid and liquid. *A, N, Q, R* are all poroelastic constants and $\rho_{11}, \rho_{12}, \rho_{22}$ are the mass coefficients following Biot (1956) such that the sums $(\rho_{11}+\rho_{12})$ and $(\rho_{12}+\rho_{22})$ are masses of solid and liquid, respectively the parameter represents mass coupling between solid and liquid. The poroelastic constants A and N corresponds to familiar have constants in a purely elastic solid. The coefficient N represents the shear modulus required on the liquid to force a certain amount of the liquid to force a certain amount of the liquid into the aggregate while the total volume remains constant. Also, the coefficient Q represents the coupling between the volume changes of solid to that of liquid. The stresses σ_{kl} and the liquid pressure *s* of the poroelastic solid are

$$\sigma_{kl}=2Ne_{kl}+(Ae+Q\varepsilon)\delta_{kl}, \qquad (k,l=r,\theta,z)$$

$$s = Qe + R\varepsilon \qquad (2)$$

where δ_{kl} is the well-known Kronecker delta function and e_{kl} are strain components of poroelastic solid.

Let (r, θ, z) be cylindrical polar co-ordinates. Consider a homogeneous, isotropic poroelastic composite hollow cylinder with inner (core) and outer (casing) shells made of different poroelastic materials and whose axis is in the direction of z-axis. The inner radius of core is r_1, outer radius of casing is r_2 and 'a' is the interface radius. The prefixes j =1, 2 are used to denote two cylinders related to poroelastic composite cylinder. The quantities with prefix (1) refer to the core, while the prefix (2) refers to the casing.

We consider the plane-strain vibrations in poroelastic cylinder such that the displacements of solid ${}_j\mathbf{u}({}_ju, 0, {}_jw)$ and liquid ${}_j\mathbf{U}({}_jU, 0, {}_jW)$ are

$$\begin{aligned} {}_ju &= \frac{\partial\, {}_j\phi_1}{\partial r} - \frac{\partial\, {}_j\psi_1}{\partial z}, \quad {}_jw = \frac{\partial\, {}_j\phi_1}{\partial z} + \frac{\partial\, {}_j\psi_1}{\partial r} + \frac{{}_j\psi_1}{r}, \\ {}_jU &= \frac{\partial\, {}_j\phi_2}{\partial r} - \frac{\partial\, {}_j\psi_2}{\partial z}, \quad {}_jW = \frac{\partial\, {}_j\phi_2}{\partial z} + \frac{\partial\, {}_j\psi_2}{\partial r} + \frac{{}_j\psi_2}{r}, \end{aligned} \tag{3}$$

where j =1, 2 and ${}_j\phi_1, {}_j\phi_2, {}_j\psi_1, {}_j\psi_2$ are functions of r,z and time t.

Substitution of Eq. (3) into Eq. (1) yields

$$\begin{aligned} {}_jP\nabla^2\, {}_j\phi_1 + {}_jQ\nabla^2\, {}_j\phi_2 &= \frac{\partial^2}{\partial t^2}({}_j\rho_{11}\, {}_j\phi_1 + {}_j\rho_{12}\, {}_j\phi_2) + b\frac{\partial}{\partial t}({}_j\phi_1 - {}_j\phi_2) \\ {}_jQ\nabla^2\, {}_j\phi_1 + {}_jR\nabla^2\, {}_j\phi_2 &= \frac{\partial^2}{\partial t^2}({}_j\rho_{12}\, {}_j\phi_1 + {}_j\rho_{22}\, {}_j\phi_2) - b\frac{\partial}{\partial t}({}_j\phi_1 - {}_j\phi_2) \\ {}_jN\nabla^2\, {}_j\psi_1 &= \frac{\partial^2}{\partial t^2}({}_j\rho_{11}\, {}_j\psi_1 + {}_j\rho_{12}\, {}_j\psi_2) + b\frac{\partial}{\partial t}({}_j\psi_1 - {}_j\psi_2) \\ 0 &= \frac{\partial^2}{\partial t^2}({}_j\rho_{12}\, {}_j\psi_1 + {}_j\rho_{22}\, {}_j\psi_2) - b\frac{\partial}{\partial t}({}_j\psi_1 - {}_j\psi_2) \end{aligned} \tag{4}$$

where ${}_jP = {}_jA + 2\, {}_jN$, ($j$ = 1, 2).

The solution of (4) can be obtained as

$$
\begin{aligned}
{}_j\phi_1 &= \left[{}_jC_1 J_0({}_j\xi_1 r) + {}_jC_2 Y_0({}_j\xi_1 r) + {}_jC_3 J_0({}_j\xi_2 r) + {}_jC_4 Y_0({}_j\xi_2 r) \right] \exp\left[i(kz+\omega t) \right], \\
{}_j\phi_2 &= -\left[{}_jC_1 \; {}_j\delta_1^2 J_0({}_j\xi_1 r) + {}_jC_2 \; {}_j\delta_1^2 Y_0({}_j\xi_1 r) + {}_jC_3 \; {}_j\delta_2^2 J_0({}_j\xi_2 r) + {}_jC_4 \; {}_j\delta_2^2 Y_0({}_j\xi_2 r) \right] \exp\left[i(kz+\omega t) \right], \\
{}_j\psi_1 &= \left[{}_jC_5 J_1({}_j\xi_3 r) + {}_jC_6 Y_1({}_j\xi_3 r) \right] \exp\left[i(kz+\omega t) \right], \\
{}_j\psi_2 &= -\frac{{}_jM_{12}}{{}_jM_{22}} \, {}_j\psi_1,
\end{aligned} \tag{5}
$$

where ${}_jC_1, {}_jC_2, {}_jC_3, {}_jC_4, {}_jC_5$ and ${}_jC_6$ are constants, ω is frequency of wave, k is wave number, J_n and Y_n are Bessel functions of first and second kind, respectively, each of order n,

$$
{}_j\xi_l^2 = \frac{\omega^2}{{}_jV_l^2} - k^2, \text{ for } l = 1,2,3,
$$

$$
{}_j\delta_k^2 = \frac{({}_jP\,{}_jR - {}_jQ^2) - {}_jV_1^2({}_jR\,{}_jM_{11} - {}_jQ\,{}_jM_{12})}{{}_jV_k^2({}_jR\,{}_jM_{12} - {}_jQ\,{}_jM_{12})} \text{ for } k = 1,2
$$

$$
{}_jM_{11} = {}_j\rho_{11} - \frac{ib}{\omega}, \; {}_jM_{12} = {}_j\rho_{12} + \frac{ib}{\omega}, \; {}_jM_{22} = {}_j\rho_{22} - \frac{ib}{\omega}, \tag{6}
$$

${}_jV_1, {}_jV_2$ are dilatational wave velocities of first and second kind, respectively, and ${}_jV_3$ is shear wave velocity.

Substituting the eqs. (5) into eqs. (3) and then using eq. (2), the displacements, stresses and liquid pressure can be obtained as

$$
\begin{aligned}
{}_j(\sigma_{rr}+s) &= [{}_jc_1 M_{11}(r) + {}_jc_2 M_{12}(r) + {}_jc_3 M_{13}(r) + {}_jc_4 M_{14}(r) + {}_jc_5 M_{15}(r) + {}_jc_6 M_{16}(r)] \exp\left[i(kz+wt) \right], \\
{}_j(\sigma_{rz}) &= [{}_jc_1 M_{21}(r) + {}_jc_2 M_{22}(r) + {}_jc_3 M_{23}(r) + {}_jc_4 M_{24}(r) + {}_jc_5 M_{25}(r) + {}_jc_6 M_{26}(r)] \exp\left[i(kz+wt) \right], \\
{}_js &= [{}_jc_1 M_{31}(r) + {}_jc_2 M_{32}(r) + {}_jc_3 M_{33}(r) + {}_jc_4 M_{34}(r) + {}_jc_5 M_{15}(r) + {}_jc_6 M_{16}(r)] \exp\left[i(kz+wt) \right], \\
{}_ju &= [{}_jc_1 M_{41}(r) + {}_jc_2 M_{42}(r) + {}_jc_3 M_{43}(r) + {}_jc_4 M_{44}(r) + {}_jc_5 M_{45}(r) + {}_jc_6 M_{46}(r)] \exp\left[i(kz+wt) \right], \\
{}_jw &= [{}_jc_1 M_{51}(r) + {}_jc_2 M_{52}(r) + {}_jc_3 M_{53}(r) + {}_jc_4 M_{54}(r) + {}_jc_5 M_{55}(r) + {}_jc_6 M_{56}(r)] \exp\left[i(kz+wt) \right], \\
{}_j\left(\frac{\partial s}{\partial r}\right) &= [{}_jc_1 N_{31}(r) + {}_jc_2 N_{32}(r) + {}_jc_3 N_{33}(r) + {}_jc_4 N_{34}(r)] \exp\left[i(kz+wt) \right]
\end{aligned} \tag{7}
$$

where the elements are

$$
\begin{aligned}
{}_jM_{11}(r) = &\left\{ \left[({}_jQ + {}_jR)\,{}_j\delta_1^2 - ({}_jA + {}_jQ) \right] k^2 + \left[({}_jQ + {}_jR)\,{}_j\delta_1^2 - ({}_jP + {}_jQ) \right] {}_j\alpha_1^2 \right\} J_0({}_j\xi_1 r) \\
&+ \frac{2\,{}_jN\,{}_j\alpha_1}{r} J_1({}_j\xi_1 r)
\end{aligned}
$$

$$
\begin{aligned}
&{}_jM_{15}(r) = -2ik\,{}_jN\,{}_j\xi_3 J_0({}_j\alpha_3 r) + \frac{2ik\,{}_jN}{r} J_1({}_j\xi_3 r)\\
&{}_jM_{21}(r) = -2ik\,{}_jN\,{}_j\alpha_1 J_1({}_j\xi_1 r)\\
&{}_jM_{25}(r) = {}_jN(k^2 - {}_j\xi_3{}^2) J_1({}_j\xi_3 r)\\
&{}_jM_{31}(r) = ({}_jR\,{}_j\delta_1{}^2 - {}_jQ)({}_j\xi_1{}^2 + k^2) J_0({}_j\xi_1 r)\\
&{}_jM_{35}(r) = 0\\
&{}_jM_{41}(r) = -{}_j\alpha_1 J_1({}_j\xi_1 r)\\
&{}_jM_{45}(r) = -ikJ_1({}_j\xi_3 r)\\
&{}_jM_{51}(r) = ikJ_0({}_j\xi_1 r)\\
&{}_jM_{55}(r) = {}_j\alpha_3 J_0({}_j\xi_3 r)
\end{aligned}
$$

${}_jM_{i2}(r) = {}_jM_{i1}(r)$ for i=1,2,3,4,5 with replacing J_n and its derivatives, respectively, by Y_n and its derivatives,

${}_jM_{i4}(r) = {}_jM_{i3}(r)$ for i=1,2,3,4,5 with replacing J_n and its derivatives, respectively, by Y_n and its derivatives,

${}_jM_{i6}(r) = {}_jM_{i5}(r)$ for i=1,2,3,4,5 with replacing J_n and its derivatives, respectively, by Y_n and its derivatives,

$$
\begin{aligned}
&{}_jN_{31}(r) = -({}_jR\,{}_j\delta_1{}^2 - {}_jQ)({}_j\xi_1{}^3 + k^2\,{}_j\xi_1) J_1({}_j\xi_1 r),\\
&{}_jN_{32}(r) = -({}_jR\,{}_j\delta_1{}^2 - {}_jQ)({}_j\xi_1{}^3 + k^2\,{}_j\xi_1) Y_1({}_j\xi_1 r),\\
&{}_jN_{33}(r) = -({}_jR\,{}_j\delta_2{}^2 - {}_jQ)({}_j\xi_2{}^3 + k^2\,{}_j\xi_2) J_1({}_j\xi_2 r),\\
&{}_jN_{34}(r) = -({}_jR\,{}_j\delta_2{}^2 - {}_jQ)({}_j\xi_2{}^3 + k^2\,{}_j\xi_2) Y_1({}_j\xi_2 r),\\
&{}_jN_{35}(r) = 0,\\
&{}_jN_{36}(r) = 0.
\end{aligned}
\tag{8}
$$

3. Boundary conditions and frequency equation:

We assume that the outer surface of casing and inner surface of core are free from stress and there is a perfect bounding at the interface, thus the boundary conditions for stress-free vibrations of a poroelastic composite hollow cylinder in case of a pervious surface are

$$at\ r = r_1;\quad {}_1(\sigma_{rr} + s) = 0,\quad {}_1(\sigma_{rz}) = 0$$

$$at\ r = r_2;\quad {}_2(\sigma_{rr} + s) = 0,\quad {}_2(\sigma_{rz}) = 0$$

$$at\ r = a;\quad {}_1\left(\sigma_{rr} + s\right) = {}_2\left(\sigma_{rr} + s\right);\quad {}_1\left(\sigma_{rz}\right) = {}_2\left(\sigma_{rz}\right);\quad {}_1u = {}_2u;\quad {}_1w = {}_2w;$$

$$at\ r = r_1, r_2 \text{ and } a\ ;\qquad {}_1s = {}_2s = 0, \tag{9}$$

while the boundary conditions in case of impervious surface are

$$at\ r = r_1;\quad {}_1(\sigma_{rr} + s) = 0,\quad {}_1(\sigma_{rz}) = 0,$$

$$at\ r = r_2;\quad {}_2(\sigma_{rr} + s) = 0,\quad {}_2(\sigma_{rz}) = 0,$$

$$at\ r = a;\quad {}_1\left(\sigma_{rr} + s\right) = {}_2\left(\sigma_{rr} + s\right);\quad {}_1\left(\sigma_{rz}\right) = {}_2\left(\sigma_{rz}\right);\quad {}_1u = {}_2u;\quad {}_1w = {}_2w;$$

$$at\ r = r_1, r_2 \text{ and } a;\quad \frac{\partial\, {}_1s}{\partial r} = \frac{\partial\, {}_2s}{\partial r} = 0\,. \tag{10}$$

Eqs. (7) and (9) results in a system of twelve homogeneous equations in constants ${}_jC_1, {}_jC_2, {}_jC_3, {}_jC_4, {}_jC_5$ and ${}_jC_6, \left(j = 1, 2\right)$ such a homogeneous system can have non-trivial solutions only if the determinant of the coefficients of the unknowns vanishes identically. Thus by eliminating the constants, the frequency equation of vibrations for poroelastic composite hollow cylinder for pervious surface is obtained as

$$\left|C_{ij}\right| = 0 \text{ for } i, j\text{=1,2,....12} \tag{11}$$

where

$C_{1j} = {}_1M_{1j}(r_1);$ j=1,2,...6,
$C_{1j} = 0;$ j=7,8,...12,
$C_{2j} = {}_1M_{2j}(r_1);$ j=1,2,...6
$C_{2j} = 0;$ j=7,8,...12,
$C_{3j} = {}_1M_{3j}(r_1);$ j=1,2,3,4, .
$C_{3j} = 0;$ j=5,6,...12,
$C_{4j} = {}_1M_{1j}(a);$ j=1,2,...6,
$C_{4j} = {}_2M_{1,j-6}(a);$ j=7,8,...12,
$C_{5j} = {}_1M_{2j}(a);$ j=1,2,...6,

$C_{5j} = {}_2M_{2,j-6}(a);$ j=7,8,...12,
$C_{6j} = {}_1M_{3j}(a);$ j=1,2,3,4,
$C_{6j} = 0\ ;$ j=5,6,...,12,

$C_{7j} = 0$; j=1,2,3,4,5,6,11,12,
$C_{7j} = {}_2M_{3,j-6}(a)$; j=7,8,9,10,
$C_{8j} = {}_1M_{4j}(a)$; j=1,2,...6,
$C_{8j} = {}_2M_{4,j-6}(a)$; j=7,8,...12,
$C_{9j} = {}_1M_{5j}(a)$; j=1,2,...6,
$C_{9j} = {}_2M_{5,j-6}(a)$; j=7,8,...12,
$C_{10,j} = 0$; j=1,2,...6,
$C_{10,j} = {}_2M_{1,j-6}(r_2)$; j=7,8,...12,

$C_{11,j} = 0$; j=1,2,...6,
$C_{11,j} = {}_2M_{2,j-6}(r_2)$; j=7,8,...12,
$C_{12,j} = 0$; j=1,2,...6, (12)
$C_{12,j} = {}_2M_{3,j-6}(r_2)$; j=7,8,...12.

In case of impervious surface, equations (7) and (9) gives the frequency equation as

$$\left|D_{ij}\right| = 0 \text{ for i, j=1,2,....12,} \tag{13}$$

where

$D_{1j} = {}_1N_{1j}(r_1)$; j=1,2,...6
$D_{1j} = 0$; j=7,8,...12;
$D_{2j} = {}_1N_{2j}(r_1)$; j=1,2,...6
$D_{2j} = 0$; j=7,8,...12;
$D_{3j} = {}_1N_{3j}(r_1)$; j=1,2,3,4
$D_{3j} = 0$; j=5,6,...12;
$D_{4j} = {}_1N_{1j}(a)$; j=1,2,...6
$D_{4j} = {}_2N_{1,j-6}(a)$; j=7,8,...12;
$D_{5j} = {}_1N_{2j}(a)$; j=1,2,...6

$D_{5j} = {}_2N_{2,j-6}(a)$; j=7,8,...12;
$D_{6j} = {}_1N_{3j}(a)$; j=1,2,3,4
$D_{6j} = 0$; j=5,6,...12;

$D_{7j} = 0$; j=1,2,3,4,5,6,11,12,
$D_{7j} = {}_2N_{3,j-6}(a)$; j=7,8,9,10,
$D_{8j} = {}_1N_{4,j}(a)$; j=1,2,...6,
$D_{8j} = {}_2N_{4,j-6}(a)$; j = 7,8,...12,
$D_{9j} = {}_1N_{5j}(a)$; j=1,2,...6,
$D_{9j} = {}_2N_{5,j-6}(a)$; j=7,8,...12,
$D_{10,j} = 0$; j=1,2,...6,
$D_{10,j} = {}_2N_{1,j-6}(r_2)$; j=7,8,...12,

$D_{11,j} = 0$; j=1,2,...6,
$D_{11,j} = {}_2N_{2,j-6}(r_2)$; j=7,8,...12,
$D_{12,j} = 0$; j=1,2,...6,
$D_{12,j} = {}_2N_{3,j-6}(r_2)$; j=7,8,...12.

and

$_1N_{lm}(r) = {}_1M_{lm}(r)$ for l=1,2,3,4,5,8,9,10,11, m=1 to 6,
$_2N_{lm}(r) = {}_2M_{lm}(r)$ for l=1,2,3,4,5,8,9,10,11, m=7 to 12, (14)
$_jN_{3m}(r)$ for m=1,2,3,4 and $_jM_{lm}(r)$ for all l, m are defined in equation (8).

Motions having infinite wavelength

When the wavelength is infinite or the wave number is zero, the frequency equation (11) of poroelastic composite hollow cylinder for pervious surface reduces to

$$A_1A_2 = 0 \qquad (15)$$

with

$$A_1 = \begin{vmatrix} {}_1M_{25} & {}_1M_{26} & 0 & 0 \\ {}_1M_{25} & {}_1M_{25} & {}_1M_{25} & {}_1M_{25} \\ {}_1M_{25} & {}_1M_{25} & {}_1M_{25} & {}_1M_{25} \\ 0 & 0 & {}_1M_{25} & {}_1M_{25} \end{vmatrix}, A_2 = \begin{vmatrix} {}_1M_{11} & {}_1M_{12} & {}_1M_{13} & {}_1M_{14} & 0 & 0 & 0 & 0 \\ {}_1M_{31} & {}_1M_{32} & {}_1M_{33} & {}_1M_{34} & 0 & 0 & 0 & 0 \\ {}_1M_{11} & {}_1M_{12} & {}_1M_{13} & {}_1M_{14} & {}_2M_{11} & {}_2M_{12} & {}_2M_{13} & {}_2M_{14} \\ {}_1M_{31} & {}_1M_{32} & {}_1M_{33} & {}_1M_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_2M_{31} & {}_2M_{32} & {}_2M_{33} & {}_2M_{34} \\ {}_1M_{41} & {}_1M_{42} & {}_1M_{43} & {}_1M_{44} & {}_2M_{41} & {}_2M_{42} & {}_2M_{43} & {}_2M_{44} \\ 0 & 0 & 0 & 0 & {}_2M_{11} & {}_2M_{12} & {}_2M_{13} & {}_2M_{14} \\ 0 & 0 & 0 & 0 & {}_2M_{31} & {}_2M_{32} & {}_2M_{33} & {}_2M_{34} \end{vmatrix} \quad (16)$$

Where the elements ${}_jM_{lm}(r)$ are defined in eqs. (12) for $k = 0$. From eq. (16) it is clear that $A_1 = 0$ or $A_2 = 0$. The frequency equation

$$A_1 = 0 \quad (17)$$

involves only shear wave velocity, hence it is the frequency equation of axially symmetric shear vibrations of a poroelastic composite hollow cylinder for infinite wavelength in case of pervious surface. The frequency equation

$$A_2 = 0 \quad (18)$$

involves only dilatational wave velocity, hence it is the frequency equation of axially symmetric dilatational vibrations of a poroelastic composite hollow cylinder for infinite wavelength in case of pervious surface. Eq.(15) shows that the axially symmetric shear and dilatational vibrations of poroelastic composite hollow cylinder for a pervious surface are uncoupled when wavelength is infinite.

Similarly, the frequency equation (13) of vibrations in poroelastic composite hollow cylinder for an impervious surface reduces to

$$B_1B_2 = 0, \quad (19)$$

with $$B_1 = \begin{vmatrix} {}_1N_{25}(r_1) & {}_1N_{26}(r_1) & 0 & 0 \\ {}_1N_{25}(a) & {}_1N_{26}(a) & {}_2N_{25}(a) & {}_2N_{26}(a) \\ {}_1N_{55}(a) & {}_1N_{56}(a) & {}_2N_{55}(a) & {}_2N_{56}(a) \\ 0 & 0 & {}_2N_{25}(r_2) & {}_2N_{26}(r_2) \end{vmatrix},$$

$$B_2 = \begin{vmatrix} {}_1N_{11}(r_1) & {}_1N_{12}(r_1) & {}_1N_{13}(r_1) & {}_1N_{14}(r_1) & 0 & 0 & 0 & 0 \\ {}_1N_{31}(r_1) & {}_1N_{32}(r_1) & {}_1N_{33}(r_1) & {}_1N_{34}(r_1) & 0 & 0 & 0 & 0 \\ {}_1N_{11}(a) & {}_1N_{12}(a) & {}_1N_{13}(a) & {}_1N_{14}(a) & {}_2N_{11}(a) & {}_2N_{12}(a) & {}_2N_{13}(a) & {}_2N_{14}(a) \\ {}_1N_{31}(a) & {}_1N_{32}(a) & {}_1N_{33}(a) & {}_1N_{34}(a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_2N_{31}(a) & {}_2N_{32}(a) & {}_2N_{33}(a) & {}_2N_{34}(a) \\ {}_1N_{41}(a) & {}_1N_{42}(a) & {}_1N_{43}(a) & {}_1N_{44}(a) & {}_2N_{41}(a) & {}_2N_{42}(a) & {}_2N_{43}(a) & {}_2N_{44}(a) \\ 0 & 0 & 0 & 0 & {}_2N_{11}(r_2) & {}_2N_{12}(r_2) & {}_2N_{13}(r_2) & {}_2N_{14}(r_2) \\ 0 & 0 & 0 & 0 & {}_2N_{31}(r_2) & {}_2N_{32}(r_2) & {}_2N_{33}(r_2) & {}_2N_{34}(r_2) \end{vmatrix} \quad (20)$$

where the elements ${}_jN_{lm}(r)$ are defined in eq. (8) and are calculated for $k = 0$.

From eq. (19) it is clear that $B_1 = 0$ or $B_2 = 0$. Equation

$$B_1=0 \quad (21)$$

is the frequency equation of axially symmetric shear vibrations of poroelastic composite hollow cylinder for an impervious surface when wavelength is infinite, whereas the equation

$$B_2=0 \quad (22)$$

is the frequency equation of dilatational vibrations of poroelastic composite hollow cylinder for an impervious surface when wavelength is infinite. Eq.(19) shows that the axially symmetric shear and dilatational vibrations of poroelastic composite hollow cylinder for an impervious surface are uncoupled. Also, we see that the equations A_1=0 and B_1=0 are same, hence the frequency equation of axially symmetric shear vibrations of poroelastic composite hollow cylinder is independent of nature of surface for infinite wavelength.

4 Particular cases

Under suitable boundary conditions the poroelastic composite hollow cylinder reduces to the following particular cases

4.1 Poroelastic composite hollow cylinder with rigid casing,
4.2 Poroelastic composite bore, and
4.2.a Poroelastic bore.

4.1 *Poroelastic composite hollow cylinder with rigid casing*

When shear modulus of the casing is larger than that of core, we can assume that casing is perfectly rigid. Letting the shear modulus of the casing approaches to infinity i.e., $N^{(2)} \to \infty$, then the shear wave velocity of casing approaches to infinity and hence $\xi_3^{(2)} \to 0$. Under this limiting condition, the frequency equation (11) of vibrations of poroelastic composite hollow cylinder for pervious surface reduces to

$$C_1\, C_2 = 0 \quad (23)$$

with $C_1 = \begin{vmatrix} {}_1M_{11}(r_1) & {}_1M_{12}(r_1) & {}_1M_{13}(r_1) & {}_1M_{14}(r_1) & {}_1M_{15}(r_1) & {}_1M_{16}(r_1) \\ {}_1M_{21}(r_1) & {}_1M_{22}(r_1) & {}_1M_{23}(r_1) & {}_1M_{24}(r_1) & {}_1M_{25}(r_1) & {}_1M_{26}(r_1) \\ {}_1M_{31}(r_1) & {}_1M_{32}(r_1) & {}_1M_{33}(r_1) & {}_1M_{34}(r_1) & 0 & 0 \\ {}_1M_{31}(a) & {}_1M_{32}(a) & {}_1M_{33}(a) & {}_1M_{34}(a) & 0 & 0 \\ {}_1M_{41}(a) & {}_1M_{42}(a) & {}_1M_{43}(a) & {}_1M_{44}(a) & {}_1M_{45}(a) & {}_1M_{46}(a) \\ {}_1M_{51}(a) & {}_1M_{52}(a) & {}_1M_{53}(a) & {}_1M_{54}(a) & {}_1M_{55}(a) & {}_1M_{56}(a) \end{vmatrix}$ and

$$C_2 = \begin{vmatrix} {}_2A_{11}(a) & {}_2A_{12}(a) & {}_2A_{13}(a) & {}_2A_{14}(a) & {}_2A_{15}(a) & {}_2A_{16}(a) \\ {}_2A_{21}(a) & {}_2A_{22}(a) & {}_2A_{23}(a) & {}_2A_{24}(a) & {}_2A_{25}(a) & {}_2A_{26}(a) \\ {}_2A_{31}(a) & {}_2A_{32}(a) & {}_2A_{33}(a) & {}_2A_{34}(a) & 0 & 0 \\ {}_2A_{11}(r_2) & {}_2A_{12}(r_2) & {}_2A_{13}(r_2) & {}_2A_{14}(r_2) & {}_2A_{15}(r_2) & {}_2A_{16}(r_2) \\ {}_2A_{21}(r_2) & {}_2A_{22}(r_2) & {}_2A_{23}(r_2) & {}_2A_{24}(r_2) & {}_2A_{25}(r_2) & {}_2A_{26}(r_2) \\ {}_2A_{31}(r_2) & {}_2A_{32}(r_2) & {}_2A_{33}(r_2) & {}_2A_{34}(r_2) & 0 & 0 \end{vmatrix} \tag{24}$$

where

$${}_2A_{11}(r) = \{({}_2Q + {}_2R)\,{}_2\alpha_1^2k^2 + ({}_2Q + {}_2R)\,{}_2\alpha_1^2 - 2\,{}_2\alpha_1^2\}J_0({}_2\xi_1 r) + \frac{2\,{}_2\alpha_1}{r}J_1({}_2\alpha_1 r),$$

$${}_2A_{12}(r) = \{({}_2Q + {}_2R)\,{}_2\alpha_1^2k^2 + ({}_2Q + {}_2R)\,{}_2\alpha_1^2 - 2\,{}_2\alpha_1^2\}Y_0({}_2\xi_1 r) + \frac{2\,{}_2\alpha_1}{r}Y_1({}_2\alpha_1 r),$$

$${}_2A_{13}(r) = \{({}_2Q + {}_2R)\,{}_2\alpha_2^2k^2 + ({}_2Q + {}_2R)\,{}_2\alpha_2^2 - 2\,{}_2\alpha_2^2\}J_0({}_2\xi_2 r) + \frac{2\,{}_2\alpha_2}{r}J_1({}_2\alpha_2 r),$$

$${}_2A_{14}(r) = \{({}_2Q + {}_2R)\,{}_2\alpha_2^2k^2 + ({}_2Q + {}_2R)\,{}_2\alpha_2^2 - 2\,{}_2\alpha_2^2\}Y_0({}_2\xi_2 r) + \frac{2\,{}_2\alpha_2}{r}Y_1({}_2\alpha_2 r),$$

$${}_2A_{15}(r) = -2ik\,{}_2\alpha_3 J_0({}_2\xi_3 r) + \frac{2ik}{r}J_1({}_2\xi_3 r),$$

$${}_2A_{16}(r) = -2ik\,{}_2\alpha_3 J_0({}_2\xi_3 r) + \frac{2ik}{r}Y_1({}_2\xi_3 r),$$

$$
\begin{aligned}
{}_2A_{21}(r) &= -2ik\,{}_2\alpha_1 J_1({}_2\xi_1 r),\\
{}_2A_{22}(r) &= -2ik\,{}_2\alpha_1 Y_1({}_2\xi_1 r),\\
{}_2A_{23}(r) &= -2ik\,{}_2\alpha_1 J_1({}_2\xi_2 r),\\
{}_2A_{24}(r) &= -2ik\,{}_2\alpha_1 Y_1({}_2\xi_2 r),\\
{}_2A_{25}(r) &= (k^2 - {}_2\xi_3^2) J_1({}_2\xi_3 r),\\
{}_2A_{26}(r) &= (k^2 - {}_2\xi_3^2) Y_1({}_2\xi_3 r),\\
{}_2A_{31}(r) &= {}_2R\ {}_2\alpha_1^2({}_2\xi_1^2 + k^2) J_0({}_2\xi_1 r),\\
{}_2A_{32}(r) &= {}_2R\ {}_2\alpha_1^2({}_2\xi_1^2 + k^2) Y_0({}_2\xi_1 r),\\
{}_2A_{33}(r) &= {}_2R\ {}_2\alpha_2^2({}_2\xi_2^2 + k^2) J_0({}_2\xi_2 r),\\
{}_2A_{34}(r) &= {}_2R\ {}_2\alpha_2^2({}_2\xi_2^2 + k^2) Y_0({}_2\xi_2 r),
\end{aligned}
$$

and

$$
{}_2\alpha_i^2 = \frac{-2\,{}_2v_i^{-2}\,{}_2R}{{}_2R\,{}_2k_{12} - {}_2Q\,{}_2k_{22}} \quad \text{for } i = 1, 2 \text{ and } 3. \tag{25}
$$

From eq. (23) it is clear that the physical parameters in the determinants C_1, C_2 are, respectively, related to core and casing. Hence, the vibrations of poroelastic composite hollow cylinder related to core and casing for pervious surface are uncoupled when the solid in casing is rigid, also we obtain $C_1= 0$ or $C_2= 0$. The equation

$$C_1 = 0, \tag{26}$$

represents the frequency equation of vibrations of poroelastic core for pervious surface when it is clamped along its outer surface, whereas the equation

$$C_2 = 0, \tag{27}$$

represents the frequency equation of vibrations of hollow rigid casing for pervious surface when the boundaries are free from stress.

In a similar way, when the solid in casing is rigid, the frequency eq. (13) of vibrations of poroelastic composite hollow cylinder for an impervious surface reduces to

$$D_1 D_2 = 0, \tag{28}$$

with $D_1 = \begin{vmatrix} {}_1N_{11}(r_1) & {}_1N_{12}(r_1) & {}_1N_{13}(r_1) & {}_1N_{14}(r_1) & {}_1N_{15}(r_1) & {}_1N_{16}(r_1) \\ {}_1N_{21}(r_1) & {}_1N_{22}(r_1) & {}_1N_{23}(r_1) & {}_1N_{24}(r_1) & {}_1N_{25}(r_1) & {}_1N_{26}(r_1) \\ {}_1N_{31}(r_1) & {}_1N_{32}(r_1) & {}_1N_{33}(r_1) & {}_1N_{34}(r_1) & 0 & 0 \\ {}_1N_{31}(a) & {}_1N_{32}(a) & {}_1N_{33}(a) & {}_1N_{34}(a) & 0 & 0 \\ {}_1N_{41}(a) & {}_1N_{42}(a) & {}_1N_{43}(a) & {}_1N_{44}(a) & {}_1N_{45}(a) & {}_1N_{46}(a) \\ {}_1N_{51}(a) & {}_1N_{52}(a) & {}_1N_{53}(a) & {}_1N_{54}(a) & {}_1N_{55}(a) & {}_1N_{56}(a) \end{vmatrix}$ and

$$D_2 = \begin{vmatrix} {}_2B_{11}(a) & {}_2B_{12}(a) & {}_2B_{13}(a) & {}_2B_{14}(a) & {}_2B_{15}(a) & {}_2B_{16}(a) \\ {}_2B_{21}(a) & {}_2B_{22}(a) & {}_2B_{23}(a) & {}_2B_{24}(a) & {}_2B_{25}(a) & {}_2B_{26}(a) \\ {}_2B_{31}(a) & {}_2B_{32}(a) & {}_2B_{33}(a) & {}_2B_{34}(a) & 0 & 0 \\ {}_2B_{11}(r_2) & {}_2B_{12}(r_2) & {}_2B_{13}(r_2) & {}_2B_{14}(r_2) & {}_2B_{15}(r_2) & {}_2B_{16}(r_2) \\ {}_2B_{21}(r_2) & {}_2B_{22}(r_2) & {}_2B_{23}(r_2) & {}_2B_{24}(r_2) & {}_2B_{25}(r_2) & {}_2B_{26}(r_2) \\ {}_2B_{31}(r_2) & {}_2B_{32}(r_2) & {}_2B_{33}(r_2) & {}_2B_{34}(r_2) & 0 & 0 \end{vmatrix} \qquad (29)$$

where

$$\begin{aligned} &{}_2B_{1m}(r) = {}_2A_{1m}(r) \text{ for } m = 1,2,3,4, \\ &{}_2B_{2m}(r) = {}_2A_{2m}(r) \text{ for } m = 1,2,3,4, \\ &{}_2B_{31}(r) = -({}_2R\ {}_2\alpha_1^{\ 2} - {}_2Q)({}_2\xi_1^{\ 3} + k^2\ {}_2\xi_1)J_1({}_2\xi_1 r), \\ &{}_2B_{32}(r) = -({}_2R\ {}_2\alpha_1^{\ 2} - {}_2Q)({}_2\xi_1^{\ 3} + k^2\ {}_2\xi_1)Y_1({}_2\xi_1 r), \\ &{}_2B_{33}(r) = -({}_2R\ {}_2\alpha_2^{\ 2} - {}_2Q)({}_2\xi_2^{\ 3} + k^2\ {}_2\xi_2)J_1({}_2\xi_2 r), \\ &{}_2B_{34}(r) = -({}_2R\ {}_2\alpha_2^{\ 2} - {}_2Q)({}_2\xi_2^{\ 3} + k^2\ {}_2\xi_2)Y_1({}_2\xi_2 r). \end{aligned} \qquad (30)$$

and ${}_jN_{lm}(r)$ are defined in eq. (14) and ${}_2A_{lm}(r)$ are defined in eq. (25).

As in the case of pervious surface, the vibrations of poroelastic composite hollow cylinder related to core and casing for an impervious surface are uncoupled when the solid in casing is rigid. From eq. (28) it is clear that $D_1 = 0$ or $D_2 = 0$. The equation

$$D_1 = 0, \qquad (31)$$

represents the frequency equation of vibrations poroelastic core for an impervious surface when it is clamped along its outer surface, whereas the equation

$$D_2 = 0, \qquad (32)$$

represents the frequency equation of vibrations of hollow rigid casing for an impervious surface when the boundaries are free from stress.

When $k = 0$, the frequency equation $C_1 = 0$ of vibrations of poroelastic core for pervious surface when it is clamped along its outer surface further reduces to

$$C_3\ C_4 = 0, \qquad (33)$$

with

$$C_3 = \begin{vmatrix} {}_1M_{11}(r_1) & {}_1M_{12}(r_1) & {}_1M_{13}(r_1) & {}_1M_{14}(r_1) \\ {}_1M_{31}(r_1) & {}_1M_{32}(r_1) & {}_1M_{33}(r_1) & {}_1M_{34}(r_1) \\ {}_1M_{31}(a) & {}_1M_{32}(a) & {}_2M_{33}(a) & {}_2M_{34}(a) \\ {}_1M_{41}(a) & {}_1M_{42}(a) & {}_1M_{43}(a) & {}_1M_{44}(a) \end{vmatrix}, \; C_4 = \begin{vmatrix} {}_1M_{25}(r_1) & {}_1M_{26}(r_1) \\ {}_1M_{55}(a) & {}_1M_{56}(a) \end{vmatrix} \qquad (34)$$

where ${}_jM_{lm}(r)$ are defined in eq. (8) and are evaluated for $k = 0$.

From eq. (33) we obtain $C_3 = 0$ or $C_4 = 0$. In particular, the equation

$$C_3 = 0, \qquad (35)$$

is the frequency equation of dilatational vibrations of poroelastic core for pervious surface when it is clamped along its outer surface in the case of infinite wavelength, while the equation

$$C_4 = 0, \qquad (36)$$

is the frequency equation of shear vibrations of poroelastic core for pervious surface when it is clamped along its outer surface in the case of infinite wavelength. Eq.(34) shows that the shear and dilatational vibrations of poroelastic core for pervious surface when it is clamped along its outer surface are uncoupled in case of infinite wavelength.

Similarly, the frequency equation $D_1 = 0$ of vibrations of poroelastic core for impervious surface when it is clamped along its outer surface reduces to

$$D_3 D_4 = 0, \qquad (37)$$

with

$$D_3 = \begin{vmatrix} {}_1N_{11}(r_1) & {}_1N_{12}(r_1) & {}_1N_{13}(r_1) & {}_1N_{14}(r_1) \\ {}_1N_{31}(r_1) & {}_1N_{32}(r_1) & {}_1N_{33}(r_1) & {}_1N_{34}(r_1) \\ {}_1N_{31}(a) & {}_1N_{32}(a) & {}_2N_{33}(a) & {}_2N_{34}(a) \\ {}_1N_{41}(a) & {}_1N_{42}(a) & {}_1N_{43}(a) & {}_1N_{44}(a) \end{vmatrix} \quad D_4 = \begin{vmatrix} {}_1N_{25}(r_1) & {}_1N_{26}(r_1) \\ {}_1N_{55}(a) & {}_1N_{56}(a) \end{vmatrix} \qquad (38)$$

where ${}_jN_{lm}(r)$ are defined in eq. (14) and are evaluated for $k = 0$.

Using eq. (37) we obtain $D_3 = 0$ or $D_4 = 0$. The equation

$$D_3 = 0, \qquad (39)$$

is the frequency equation of dilatational vibrations of poroelastic core for impervious surface when it is clamped along its outer surface in the case of infinite wavelength, while the equation

$$D_4 = 0, \qquad (40)$$

is the frequency equation of shear vibrations of poroelastic core for impervious surface when it is clamped along its outer surface in the case of infinite wavelength. This equation is same as eq. (35), hence the frequency equation of shear vibrations of poroelastic core when it is clamped along its outer surface in the case of infinite wavelength is independent of nature of surface. In addition, eq. (37) shows that the shear and dilatational vibrations of poroelastic core for impervious surface when it is clamped along its outer surface are uncoupled.

When $k = 0$, the frequency equation $C_2 = 0$ of vibrations of poroelastic casing for pervious surface when the solid is rigid further reduces to

$$C_5\, C_6 = 0, \tag{41}$$

with

$$C_5 = \begin{vmatrix} {}_2A_{11}(a) & {}_2A_{12}(a) & {}_2A_{13}(a) & {}_2A_{14}(a) \\ {}_2A_{31}(a) & {}_2A_{32}(a) & {}_2A_{33}(a) & {}_2A_{34}(a) \\ {}_2A_{11}(r_2) & {}_2A_{12}(r_2) & {}_2A_{13}(r_2) & {}_2A_{14}(r_2) \\ {}_2A_{31}(r_2) & {}_2A_{32}(r_2) & {}_2A_{33}(r_2) & {}_2A_{34}(r_2) \end{vmatrix}, C_6 = \begin{vmatrix} {}_2A_{25}(a) & {}_2A_{26}(a_1) \\ {}_2A_{25}(r_2) & {}_2A_{26}(r_2) \end{vmatrix} \tag{42}$$

where ${}_jA_{lm}(r)$ are defined in eq. (25) and are evaluated for $k = 0$.

From eq. (41) we obtain $C_5 = 0$ or $C_6 = 0$. In particular, the equation

$$C_5 = 0, \tag{43}$$

is the frequency equation of dilatational vibrations of poroelastic casing for pervious surface when the solid is rigid in the case of infinite wavelength, while the equation

$$C_6 = 0, \tag{44}$$

is the frequency equation of shear vibrations of poroelastic casing for pervious surface when the solid is rigid in the case of infinite wavelength. Eq.(41) shows that the shear and dilatational vibrations of poroelastic casing for pervious surface when the solid is rigid are uncoupled in the case of infinite wavelength.

Similarly, the frequency equation $D_1 = 0$ of vibrations of poroelastic casing for impervious surface when the solid is rigid reduces to

$$D_5\, D_6 = 0, \tag{45}$$

with

$$D_5 = \begin{vmatrix} {}_2B_{11}(a) & {}_2B_{12}(a) & {}_2B_{13}(a) & {}_2B_{14}(a) \\ {}_2B_{31}(a) & {}_2B_{32}(a) & {}_2B_{33}(a) & {}_2B_{34}(a) \\ {}_2B_{11}(r_2) & {}_2B_{12}(r_2) & {}_2B_{13}(r_2) & {}_2B_{14}(r_2) \\ {}_2B_{31}(r_2) & {}_2B_{32}(r_2) & {}_2B_{33}(r_2) & {}_2B_{34}(r_2) \end{vmatrix}, D_6 = \begin{vmatrix} {}_2B_{25}(a) & {}_2B_{26}(a_1) \\ {}_2B_{25}(r_2) & {}_2B_{26}(r_2) \end{vmatrix} \tag{46}$$

where ${}_jB_{lm}(r)$ are defined in eq. (30) and are evaluated for $k = 0$.

Using eq. (45) we obtain $D_5 = 0$ or $D_6 = 0$. The equation

$$D_5 = 0, \tag{47}$$

is the frequency equation of dilatational vibrations of poroelastic casing for impervious surface when the solid is rigid in the case of infinite wavelength, while the equation

$$D_6 = 0, \tag{48}$$

is the frequency equation of shear vibrations of poroelastic casing for impervious surface when the solid is rigid in the case of infinite wavelength. This equation is same as eq. (44), hence the frequency equation of shear vibrations of poroelastic casing for impervious surface when the solid is rigid in the case of infinite wavelength is independent of nature of surface. In addition, eq. (45) shows that the shear and dilatational vibrations of poroelastic casing for impervious surface when the solid is rigid are uncoupled in the case of infinite wavelength.

4.2 *Poroelastic composite bore*

When the outer radius r_2 of casing tends to ∞, the frequency equation (11) of poroelastic composite hollow cylinder for pervious surface reduces to

$$E_1 = 0, \tag{49}$$

where

$$E_1 = \begin{vmatrix} {}_1M_{11}(r_1) & {}_1M_{12}(r_1) & {}_1M_{13}(r_1) & {}_1M_{14}(r_1) & {}_1M_{15}(r_1) & {}_1M_{16}(r_1) & 0 & 0 & 0 \\ {}_1M_{21}(r_1) & {}_1M_{22}(r_1) & {}_1M_{23}(r_1) & {}_1M_{24}(r_1) & {}_1M_{25}(r_1) & {}_1M_{26}(r_1) & 0 & 0 & 0 \\ {}_1M_{31}(r_1) & {}_1M_{32}(r_1) & {}_1M_{33}(r_1) & {}_1M_{34}(r_1) & 0 & 0 & 0 & 0 & 0 \\ {}_1M_{11}(a) & {}_1M_{12}(a) & {}_1M_{13}(a) & {}_1M_{14}(a) & {}_1M_{15}(a) & {}_1M_{16}(a) & {}_2M_{12}(a) & {}_2M_{14}(a) & {}_2M_{16}(a) \\ {}_1M_{21}(a) & {}_1M_{22}(a) & {}_1M_{23}(a) & {}_1M_{24}(a) & {}_1M_{25}(a) & {}_1M_{26}(a) & {}_2M_{22}(a) & {}_2M_{24}(a) & {}_2M_{26}(a) \\ {}_1M_{31}(a) & {}_1M_{32}(a) & {}_1M_{33}(a) & {}_1M_{34}(a) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & {}_2M_{32}(a) & {}_2M_{34}(a) & {}_2M_{36}(a) \\ {}_1M_{41}(a) & {}_1M_{42}(a) & {}_1M_{43}(a) & {}_1M_{44}(a) & {}_1M_{45}(a) & {}_1M_{46}(a) & {}_2M_{42}(a) & {}_2M_{44}(a) & {}_2M_{46}(a) \\ {}_1M_{51}(a) & {}_1M_{52}(a) & {}_1M_{53}(a) & {}_1M_{54}(a) & {}_1M_{55}(a) & {}_1M_{56}(a) & {}_2M_{52}(a) & {}_2M_{54}(a) & {}_2M_{56}(a) \end{vmatrix}$$

where the elements ${}_jM_{\mathrm{lm}}$ are defined in eq. (8).

Eq. (49) is the frequency equation of poroelastic composite bore for a pervious surface.

Similarly, the frequency equation of poroelastic composite bore for impervious surface can be obtained as

$$F_1 = 0, \tag{50}$$

where

$$F_1 = \begin{vmatrix} {}_1N_{11}(r_1) & {}_1N_{12}(r_1) & {}_1N_{13}(r_1) & {}_1N_{14}(r_1) & {}_1N_{15}(r_1) & {}_1N_{16}(r_1) & 0 & 0 & 0 \\ {}_1N_{21}(r_1) & {}_1N_{22}(r_1) & {}_1N_{23}(r_1) & {}_1N_{24}(r_1) & {}_1N_{25}(r_1) & {}_1N_{26}(r_1) & 0 & 0 & 0 \\ {}_1N_{31}(r_1) & {}_1N_{32}(r_1) & {}_1N_{33}(r_1) & {}_1N_{34}(r_1) & 0 & 0 & 0 & 0 & 0 \\ {}_1N_{11}(a) & {}_1N_{12}(a) & {}_1N_{13}(a) & {}_1N_{14}(a) & {}_1N_{15}(a) & {}_1N_{16}(a) & {}_2N_{12}(a) & {}_2N_{14}(a) & {}_2N_{16}(a) \\ {}_1N_{21}(a) & {}_1N_{22}(a) & {}_1N_{23}(a) & {}_1N_{24}(a) & {}_1N_{25}(a) & {}_1N_{26}(a) & {}_2N_{22}(a) & {}_2N_{24}(a) & {}_2N_{26}(a) \\ {}_1N_{31}(a) & {}_1N_{32}(a) & {}_1N_{33}(a) & {}_1N_{34}(a) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & {}_2N_{32}(a) & {}_2N_{34}(a) & {}_2N_{36}(a) \\ {}_1N_{41}(a) & {}_1N_{42}(a) & {}_1N_{43}(a) & {}_1N_{44}(a) & {}_1N_{45}(a) & {}_1N_{46}(a) & {}_2N_{42}(a) & {}_2N_{44}(a) & {}_2N_{46}(a) \\ {}_1N_{51}(a) & {}_1N_{52}(a) & {}_1N_{53}(a) & {}_1N_{54}(a) & {}_1N_{55}(a) & {}_1N_{56}(a) & {}_2N_{52}(a) & {}_2N_{54}(a) & {}_2N_{56}(a) \end{vmatrix} \tag{51}$$

Eq. (50) is the frequency equation of poroelastic composite bore for an impervious surface.

where the elements ${}_jN_{\mathrm{lm}}$ are defined in eq. (14).

For infinite wavelength, the frequency equation $E_1 = 0$ of vibrations of poroelastic composite bore for pervious surface reduces to

$$E_2\, E_3 = 0, \tag{52}$$

with

$$E_2 = \begin{vmatrix} {}_1M_{25}(r_1) & {}_1M_{26}(r_1) & 0 \\ {}_1M_{25}(a) & {}_1M_{26}(a) & {}_2M_{26}(a) \\ {}_1M_{55}(a) & {}_1M_{56}(a) & {}_2M_{56}(a) \end{vmatrix} \text{ and}$$

$$E_3 = \begin{vmatrix} {}_1M_{11}(r_1) & {}_1M_{12}(r_1) & {}_1M_{13}(r_1) & {}_1M_{14}(r_1) & 0 & 0 \\ {}_1M_{31}(r_1) & {}_1M_{32}(r_1) & {}_1M_{33}(r_1) & {}_1M_{34}(r_1) & 0 & 0 \\ {}_1M_{11}(a) & {}_1M_{12}(a) & {}_1M_{13}(a) & {}_1M_{14}(a) & {}_2M_{12}(a) & {}_2M_{14}(a) \\ {}_1M_{31}(a) & {}_1M_{32}(a) & {}_1M_{33}(a) & {}_1M_{34}(a) & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_2M_{32}(a) & {}_2M_{34}(a) \\ {}_1M_{51}(a) & {}_1M_{52}(a) & {}_1M_{53}(a) & {}_1M_{54}(a) & {}_2M_{52}(a) & {}_2M_{54}(a) \end{vmatrix} \tag{53}$$

where the elements ${}_jM_{\mathrm{lm}}$ are defined in eq. (8) and are evaluated for $k = 0$.

From eq. (52), it is clear that $E_2 = 0$ or $E_3 = 0$. In particular,

$$E_2 = 0, \tag{54}$$

is the frequency equation of shear vibrations of poroelastic composite bore for pervious surface, whereas the equation

$$E_3 = 0, \tag{55}$$

is the frequency equation of dilatational vibrations of poroelastic composite bore for pervious surface. Eq. (50) shows that the shear vibrations and dilatational vibrations of poroelastic composite bore for pervious surface are uncoupled.

In a similar way, for infinite wavelength, the frequency equation $F_1 = 0$ of vibrations of poroelastic composite bore for an impervious surface reduces to

$$F_2 F_3 = 0, \tag{56}$$

with

$$F_2 = \begin{vmatrix} {}_1N_{25}(r_1) & {}_1N_{26}(r_1) & 0 \\ {}_1N_{25}(a) & {}_1N_{26}(a) & {}_2N_{26}(a) \\ {}_1N_{55}(a) & {}_1N_{56}(a) & {}_2N_{56}(a) \end{vmatrix} \text{ and}$$

$$F_3 = \begin{vmatrix} {}_1N_{11}(r_1) & {}_1N_{12}(r_1) & {}_1N_{13}(r_1) & {}_1N_{14}(r_1) & 0 & 0 \\ {}_1N_{31}(r_1) & {}_1N_{32}(r_1) & {}_1N_{33}(r_1) & {}_1N_{34}(r_1) & 0 & 0 \\ {}_1N_{11}(a) & {}_1N_{12}(a) & {}_1N_{13}(a) & {}_1N_{14}(a) & {}_2N_{12}(a) & {}_2N_{14}(a) \\ {}_1N_{31}(a) & {}_1N_{32}(a) & {}_1N_{33}(a) & {}_1N_{34}(a) & 0 & 0 \\ 0 & 0 & 0 & 0 & {}_2N_{32}(a) & {}_2N_{34}(a) \\ {}_1N_{51}(a) & {}_1N_{52}(a) & {}_1N_{53}(a) & {}_1N_{54}(a) & {}_2N_{52}(a) & {}_2N_{54}(a) \end{vmatrix} \tag{57}$$

where the elements ${}_jN_{lm}$ are defined in eq. (14) and are evaluated for $k = 0$.

From eq. (56), clearly $F_2 = 0$ or $F_3 = 0$. In particular,

$$F_2 = 0, \tag{58}$$

is the frequency equation of shear vibrations of poroelastic composite bore for an impervious surface which is same as equation E_2=0 using (14), hence the frequency equation of shear vibrations of poroelastic composite bore is independent of nature of surface for infinite wavelength.

The equation

$$F_3 = 0, \tag{59}$$

is the frequency equation of dilatational vibrations of poroelastic composite bore for an impervious surface. Eq. (56) shows that the shear vibrations and dilatational vibrations of poroelastic composite bore for an impervious surface are uncoupled for infinite wavelength.

4.2.a ***Poroelastic bore***

When the material parameters of core and casing are same i.e. ${}_2P = {}_1P = P,\ {}_2Q = {}_1Q = Q,\ {}_2R = {}_1R = R$ and ${}_2N = {}_1N = N$ then the poroelastic composite bore will become a poroelastic bore of radius r_1. Under these conditions the frequency equation (48) of poroelastic composite bore for pervious surface reduces to

$$G_1 = 0, \tag{60}$$

with
$$G_1 = \begin{vmatrix} {}_1M_{12}(r_1) & {}_1M_{14}(r_1) & {}_1M_{16}(r_1) \\ {}_1M_{22}(r_1) & {}_1M_{24}(r_1) & {}_1M_{26}(r_1) \\ {}_1M_{32}(r_1) & {}_1M_{34}(r_1) & 0 \end{vmatrix}, \tag{61}$$

where the elements ${}_jM_{\mathrm{lm}}$ are defined in eq. (8).

Eq. (62) is the frequency equation of poroelastic bore for a pervious surface.

Similarly, the frequency equation of poroelastic bore for impervious surface can be obtained as

$$H_1 = 0, \tag{62}$$

with
$$H_1 = \begin{vmatrix} {}_1N_{12}(r_1) & {}_1N_{14}(r_1) & {}_1N_{16}(r_1) \\ {}_1N_{22}(r_1) & {}_1N_{24}(r_1) & {}_1N_{26}(r_1) \\ {}_1N_{32}(r_1) & {}_1N_{34}(r_1) & 0 \end{vmatrix}, \tag{63}$$

where the elements ${}_jN_{\mathrm{lm}}$ are defined in eq. (14).

Eq. (62) is the frequency equation of poroelastic bore for an impervious surface.

5. Non-dimensionalization of frequency equation

The natural frequency will be real when the dissipation coefficient is zero i.e. $b = 0$. For the sake of numerical work the dissipation coefficient 'b' is taken as zero and hence we obtained only real frequency. To analyze the frequency equations of plane-strain vibrations of poroelastic composite hollow cylinders, it is convenient to introduce the following non-dimensional parameters:

$$a_1 = \frac{{}_2P}{{}_1H},\ a_2 = \frac{{}_2Q}{{}_1H},\ a_3 = \frac{{}_2R}{{}_1H},\ a_4 = \frac{{}_2N}{{}_1H},\ d_1 = \frac{{}_2\rho_{11}}{{}_1\rho},\ d_2 = \frac{{}_2\rho_{12}}{{}_1\rho},\ d_3 = \frac{{}_2\rho_{22}}{{}_1\rho},$$
$$b_1 = \frac{{}_1P}{{}_1H},\ b_2 = \frac{{}_1Q}{{}_1H},\ b_3 = \frac{{}_1R}{{}_1H},\ b_4 = \frac{{}_1N}{{}_1H},\ g_1 = \frac{{}_1\rho_{11}}{{}_1\rho},\ g_2 = \frac{{}_1\rho_{12}}{{}_1\rho},\ g_3 = \frac{{}_1\rho_{22}}{{}_1\rho},$$
$$x_1 = (\frac{{}_1V_0}{{}_1V_1})^2,\ y_1 = (\frac{{}_1V_0}{{}_1V_2})^2,\ z_1 = (\frac{{}_1V_0}{{}_1V_3})^2, x_2 = (\frac{{}_1V_0}{{}_2V_1})^2,\ y_2 = (\frac{{}_1V_0}{{}_2V_2})^2,\ z_2 = (\frac{{}_1V_0}{{}_2V_3})^2,\ \Omega = \frac{\omega h}{{}_1C_0}, \tag{64}$$

where Ω is non-dimensional frequency and

$${}_1H = {}_1P + 2{}_1Q + {}_1R,\quad {}_1\rho = {}_1\rho_{11} + 2{}_1\rho_{12} + {}_1\rho_{22},\quad {}_1C_0^2 = \frac{{}_1N}{{}_1\rho},\quad {}_1V_0^2 = \frac{{}_1H}{{}_1\rho}. \tag{65}$$

Non-dimensional frequency (Ω) is calculated for two types of composite cylinders, namely composite cylinder-I and composite cylinder-II for each pervious and impervious surface. Composite cylinder-I consists of core made up of sandstone saturated with water (Yew and Jogi, 1976) and casing is made up of sandstone saturated with kerosene

(Fatt, 1957), where as in composite cylinder-II, the core is sandstone saturated with kerosene and casing is sandstone saturated with water. The physical parameters of these poroelastic composite materials following equation (64) are given in Table 1.

Table - 1

Material Parameters	a_1	a_2	a_3	a_4	d_1	d_2	d_3	x_2	y_2	z_2
Composite Cylinder-I	0.445	0.034	0.015	0.123	0.887	-0.001	0.099	1.863	8.884	7.183
Composite Cylinder–II	1.819	0.011	0.054	0.780	0.891	0	0.125	0.489	2.330	1.142

b_1	b_2	b_3	b_4	g_1	g_2	g_3	x_1	y_1	z_1
0.960	0.006	0.028	0.412	0.877	0	0.123	0.913	4.347	2.129
0.843	0.065	0.028	0.234	0.901	-0.001	0.101	0.999	4.763	3.851

6. Results and Discussion

For a given poroelastic parameters, frequency equations when non-dimensionalized using equation (64), constitute a relation between non-dimensional phase velocity and wave number. Different values of r_1/a and r_2/a, viz., 1.1 and 3 are taken for numerical computation. These values, respectively, represent thin and thick poroelastic shells.

Figs. 1-4 depict phase velocity of vibrations of poroelastic composite hollow cylinders I and II for different combinations of thin and thick shells for pervious and impervious surfaces. In Fig.1, phase velocity for thin core and thin casing is has been plotted. It is clear that the phase velocity of composite cylinder I is more than that of cylinder II and the phase velocity of cylinder II is steady. Also, there is no much difference between phase velocities of pervious and impervious surfaces for each of the cylinders I and II. Fig. 2 shows the phase velocity for thin core and thick casings. The phase velocity is same for pervious and impervious surfaces when the wavenumber is < 1 and > 4 in case of cylinder I, whereas in case of cylinder II it is true when the wavenumber is <1 and >7. The variation of phase velocity for thin casing and thick core is shown in Fig. 3. The phase velocity is same for pervious and impervious surfaces when the wavenumber is < 4 and > 6 in case of cylinder I, whereas in case of cylinder II it is true when the wavenumber is < 2 and >7.3. Fig. 4 shows the phase velocity for thick core and thick casings. The phase velocity is same for pervious and impervious surfaces when the wavenumber is < 7 in case of cylinder I, whereas in case of cylinder II the phase velocity is different for pervious and impervious surfaces.

The variation in phase velocity for poroelastic core when it is clamped along its outer surface is shown in Figs. 5-6. In particular, thin core is considered in Fig.5, whereas thick core is considered in Fig.6. From Fig. 5, it is clear that the phase velocity is same for both cylinders for each pervious and impervious surface. The phase velocity is same for both cylinders when wave number is between 0 and 2. Also, the phase velocity is maximum when wavenumber is 1. In case of thick core, phase velocity is same for both cylinders when wave number is between 0 and 1.

Figs. 7-8 depict phase velocity for poroelastic casing when the solid is rigid. In particular, thin casing is considered in Fig.7, whereas thick casing is considered in Fig. 8. In case of thin casing, the phase velocity is more for pervious surface than that of impervious surface for both cylinders I and II. In particular, the phase velocity is steady for impervious surface for both the cylinders. The maximum phase velocity is observed when wavenumber is 7 for impervious surface for cylinder I in case of thick casing.

The variation in phase velocity for poroelastic composite bore is shown in Figs. 9-10. In particular, composite bore with thin core is considered in Fig. 9, whereas composite bore with thick core is considered in Fig.10. From Fig. 9, it is clear that the phase velocity for cylinder I higher than that of cylinder II when wavenumber is < 4.5 for impervious surface. Also, the maximum phase velocity is observed when wavenumber is 8 for impervious surface for cylinder-I.

7. Conclusion:

The study of vibrations in poroelastic composite hollow cylinders and bore has lead to the following conclusions:

(i) The axially symmetric shear and dilatational vibrations of poroelastic composite hollow cylinder for a pervious surface are uncoupled when wavelength is infinite.
(ii) The frequency equation of axially symmetric shear vibrations of poroelastic composite hollow cylinder is independent of nature of surface for infinite wavelength.
(iii) The vibrations of poroelastic composite hollow cylinder related to core and casing for pervious surface are uncoupled when the solid in casing is rigid. In particular, in case of core the vibrations are observed when it is clamped along its outer surface.
(iv) The shear and dilatational vibrations of poroelastic core for pervious surface when it is clamped along its outer surface are uncoupled in case of infinite wavelength.
(v) The shear and dilatational vibrations of poroelastic casing for pervious surface when the solid is rigid are uncoupled in the case of infinite wavelength.
(vi) The frequency equation of shear vibrations of poroelastic casing is independent of nature of surface when the solid is rigid in the case of infinite wavelength

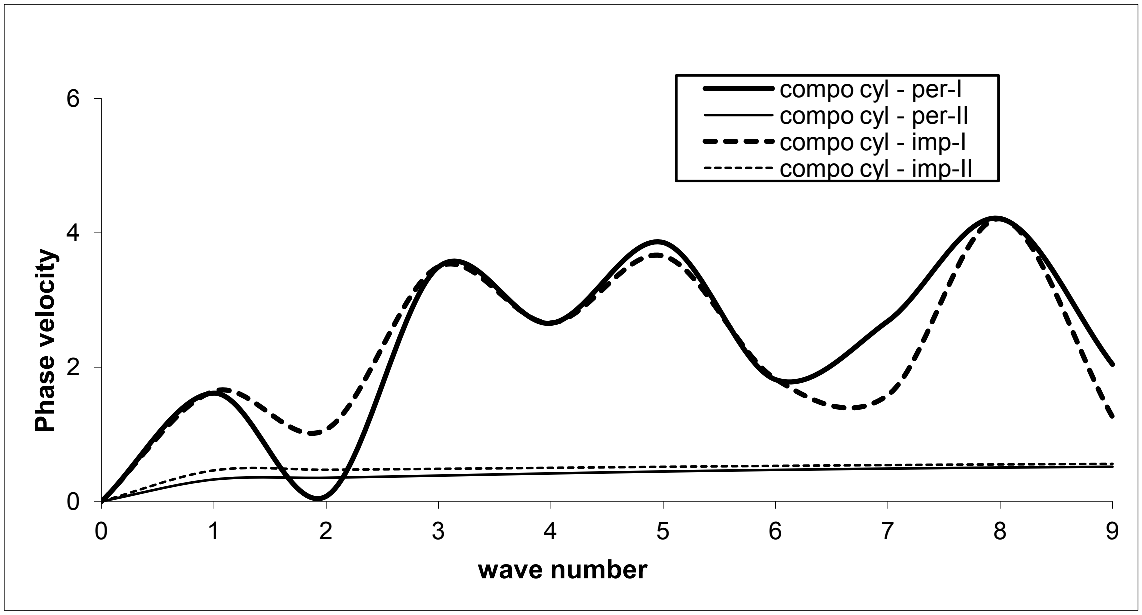

Fig.1 Variation of phase velocity with the wave number – *poroelastic composite hollow cylinder* – Thin core and thin casing

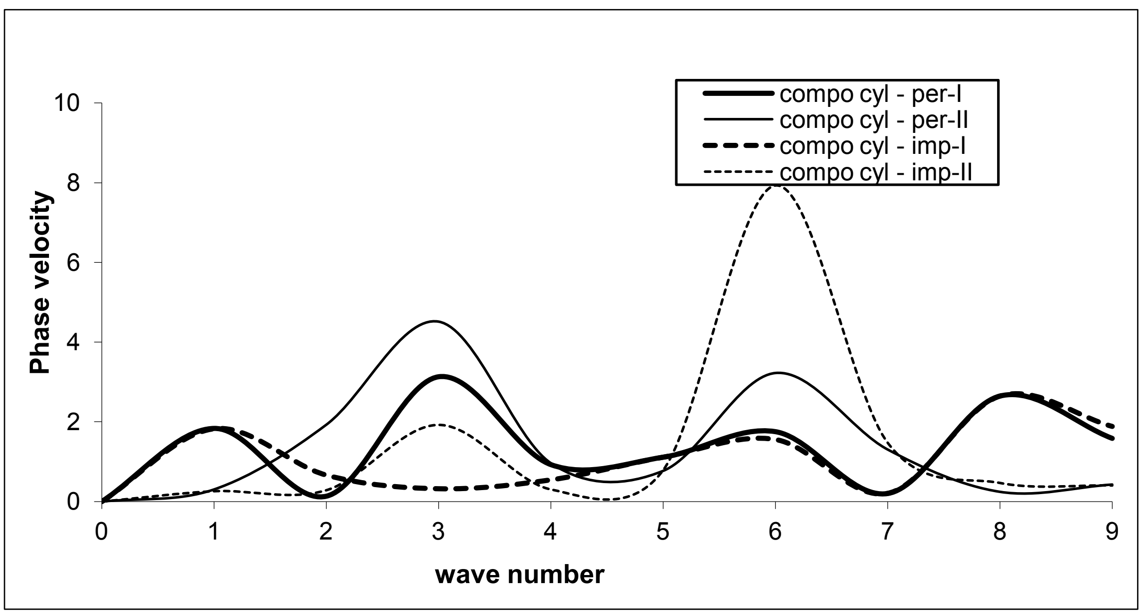

Fig. 2 Variation of phase velocity with the wave number – *poroelastic composite hollow cylinder* – Thin core and thik casing

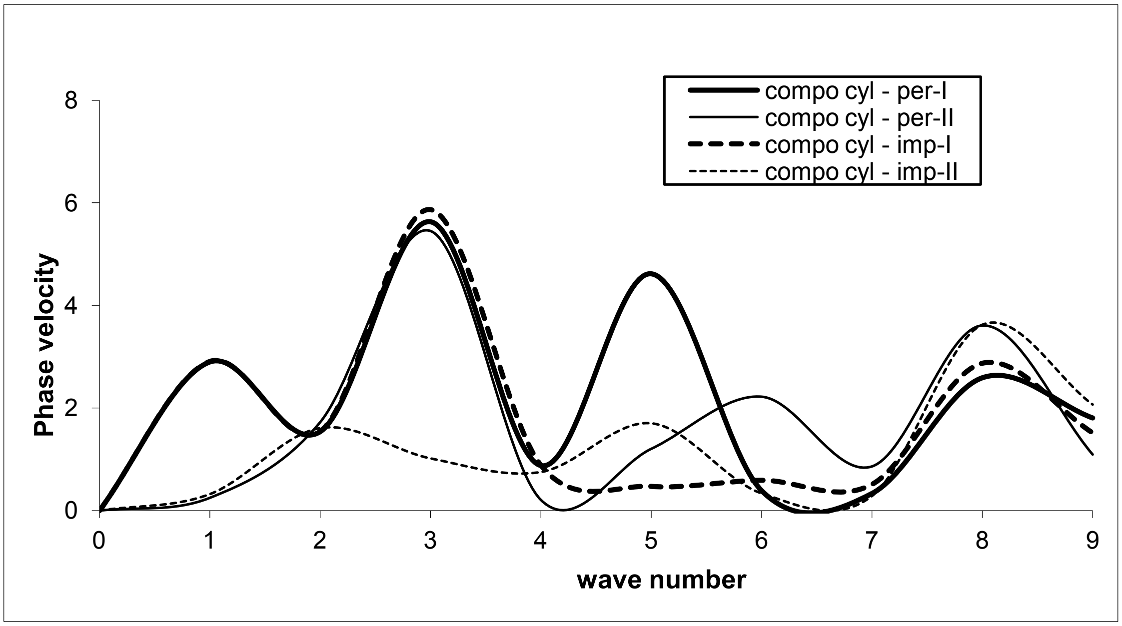

Fig.3 Variation of phase velocity with the wave number – *poroelastic composite hollow cylinder* – Thick core and thin casing

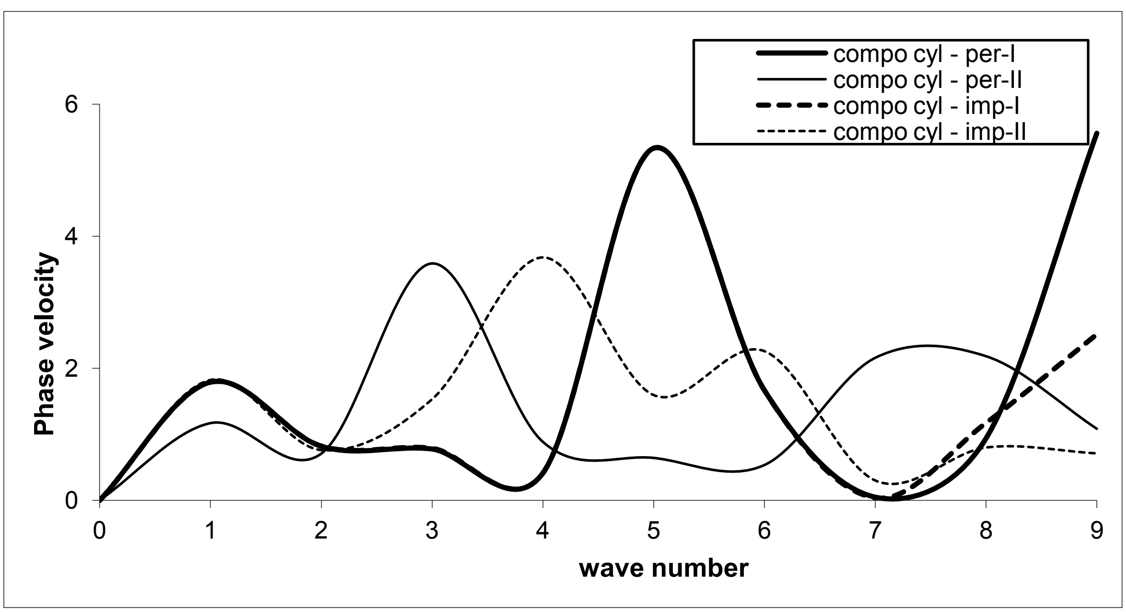

Fig.4 Variation of phase velocity with the wave number – *poroelastic composite hollow cylinder* – Thick core and thick casing

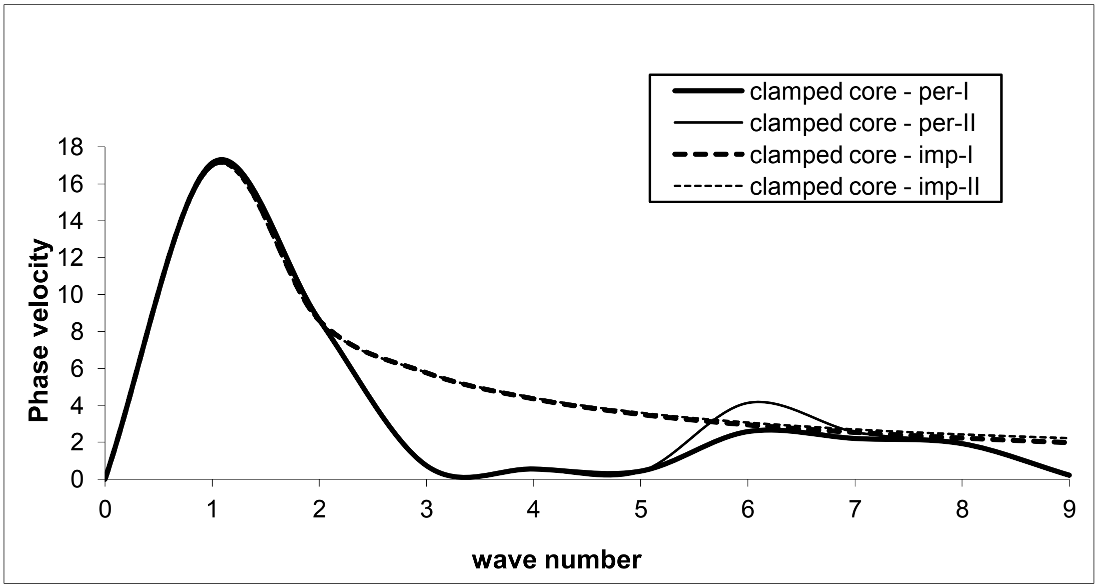

Fig.5 Variation of phase velocity with the wave number – *poroelastic shell clamped along its outer surface* Thin shell

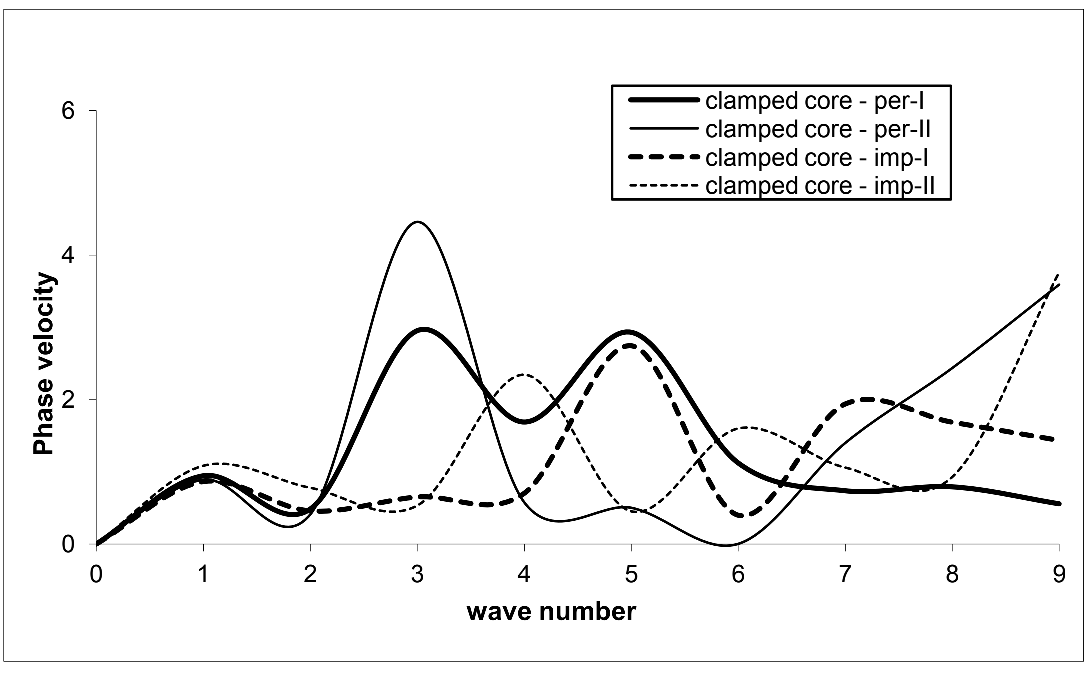

Fig.6 Variation of phase velocity with the wave number – *poroelastic shell clamped along its outer surface* – Thick shell

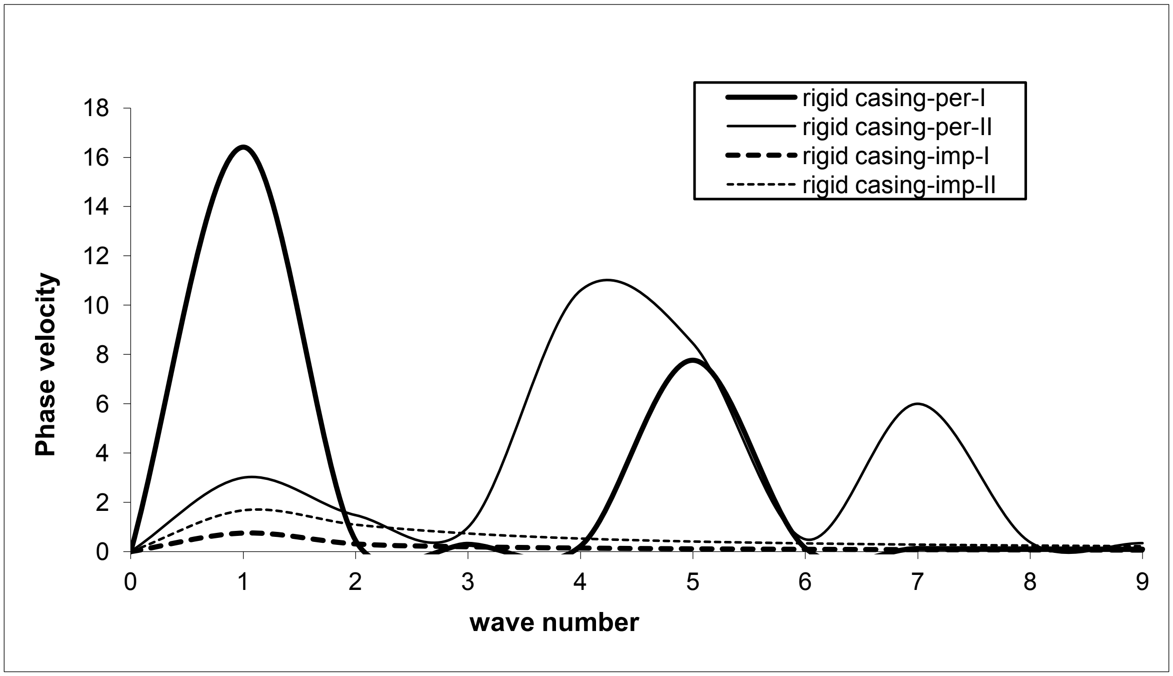

Fig.7 Variation of phase velocity with the wave number – *poroelastic rigid casingl* – Thin shell

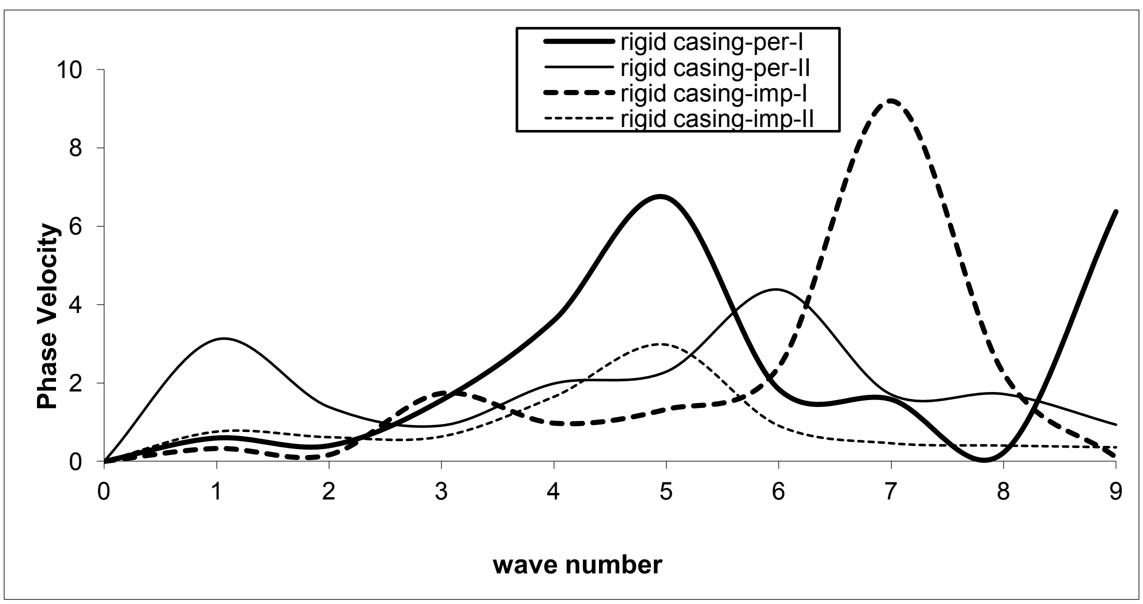

Fig.8 Variation of phase velocity with the wave number – *poroelastic rigid casingl* – Thick shell

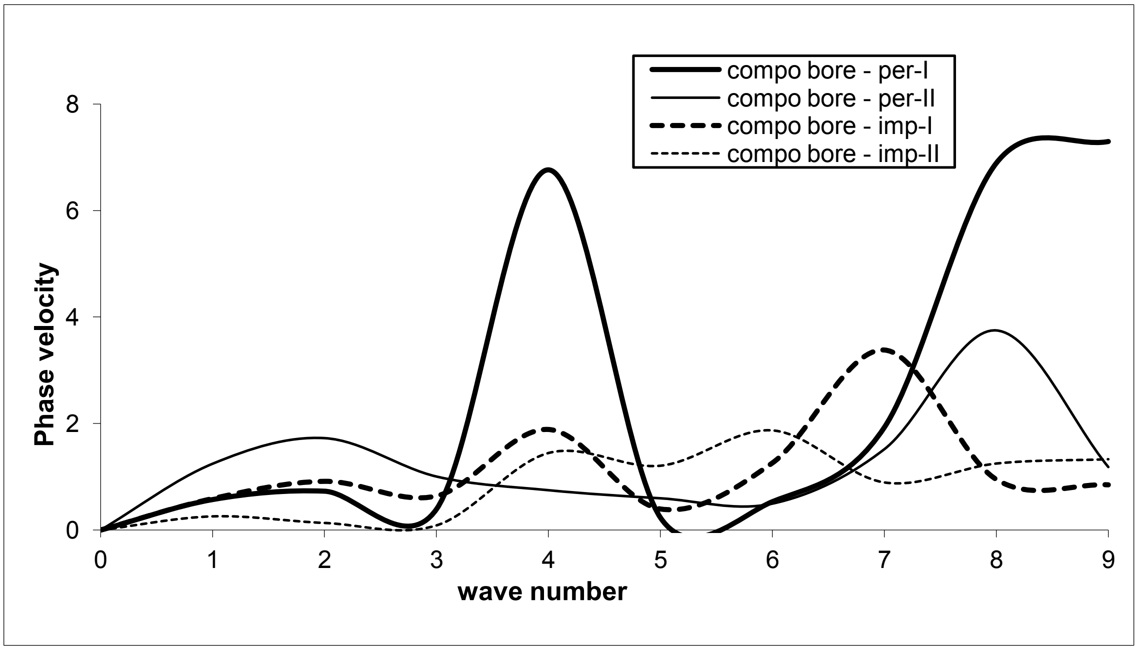

Fig.9 Variation of phase velocity with the wave number – *poroelastic composite bore* – Thin core

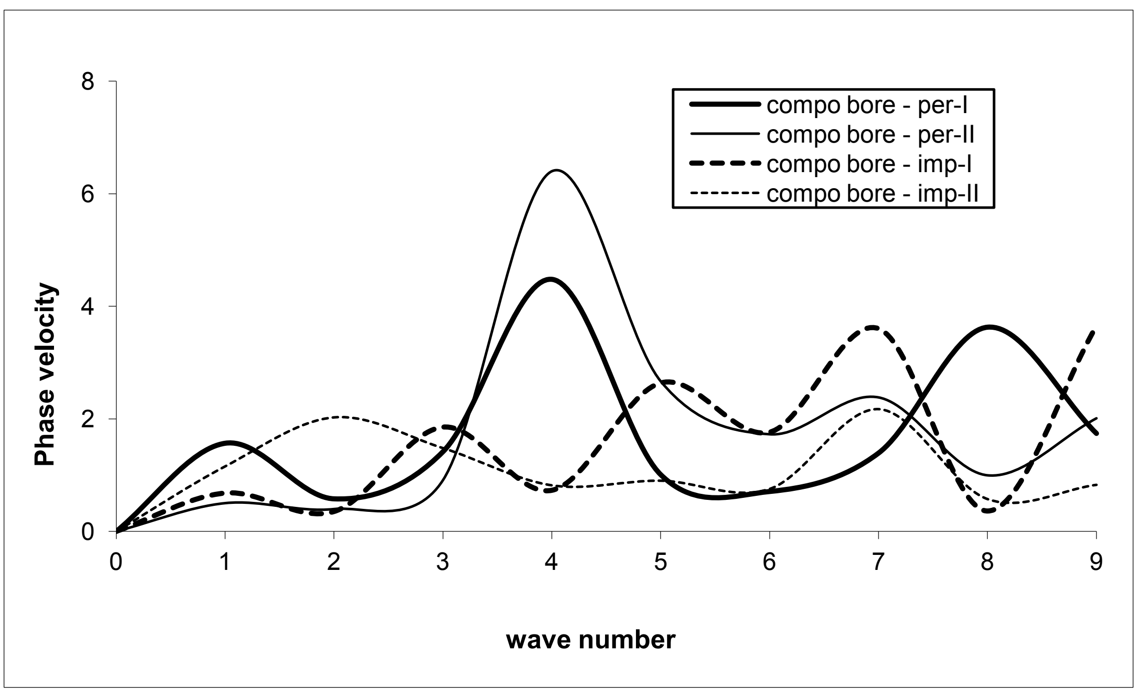

Fig.10 Variation of phase velocity with the wave number – *poroelastic composite bore* – Thick core

References

1. Abousleiman, Y.N., and Cui. L. 1998. Poroelastic solutions in transversely isotropic media for wellbore cylinders, Int. J. Solids Struct., 35, 4905-4929.
2. Ahmed Shah, S. 2011. Flexural wave propagation in coated poroelastic cylinders with reference to fretting fatigue, Journal of vibration and control, 17, 1049-1064.
3. Ahmed Shah, S. and Tajuddin, M. 2009. Axially symmetric vibrations of finite composite poroelastic cylinders, Int. J. of Applied Mechanics and Engineering, 14, 865-877.
4. Biot, M.A. 1956. The theory of propagation of elastic waves in fluid-saturated porous solid, J. Acous. Soc. Am., 28, 168-178.
5. Cui, L. Cheng A. H. D. and Abousleiman Y. 1997. Poroelastic solutions of an inclined Borehole – Trans. ASME, J. Appl. Mech., 64, 32-38.
6. Fatt, I. 1959. The Biot-willis elastic coefficients for a sandstone, J. Appl. Mech., 26, 296-297.
7. Gazis D.C. 1958. **Exact analysis of** Plane-strain vibrations in thick-walled hollow cylinders, J.Acoust. Soc. Am., 30, 786-794.
8. Malla Reddy, P. and Tajuddin, M. 2010. Axially symmetric vibrations of composite poroelastic cylinders in the context of fretting fatigue, Special Topics and Reviews in Porous Media, 1, 311-320.
9. McNiven H.D. Sackman J.L. and Shah A.H. 1963. Dispersion of axially symmetric waves in composite elastic rods, J.Acoust. Soc. Am., 35, 1602-1609.
10. Shanker.B, Manoj Kumar. J, Ahmed Shah .S, and Nageswara Nath .C 2012. Radial vibrations of an infinitely long poroelastic composite hollow circular cylinder, International Journal of Engineering, Science and Technology, 4, 17-33.
11. Tajuddin, M. Nageswara Nath, C. and Manoj Kumar, J. 2011. Axial-shear vibrations of an infinitely long poroelastic composite circular cylinder, Special Topics & Reviews in Porous Media, 2,.133-143.
12. Yew, C.H. and Jogi, P.N. 1976. Study of wave motions in fluid-saturated porous rocks, J. Acoust. Soc. Am., 60, 2-8.